II. *A Comparison of Platinum and Gas Thermometers, including a Determination of the Boiling-Point of Sulphur on the Nitrogen Scale. An Account of Experiments made in the Laboratory of the Bureau International des Poids et Mesures at Sèvres.*

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[Plate 1.]

Contents.

I. Introduction.

In a paper entitled "The Practical Measurement of Temperature," read before the Royal Society in 1886, Professor Callendar drew attention to the method of measuring temperature based on the determination of the electrical resistance of a platinum wire. He showed that the method was capable of a very general application, and that the platinum resistance thermometer was an instrument giving consistent and accurate results over a very wide temperature range.

Callendar pointed out that if R_0 denote the resistance of the spiral of a particular platinum thermometer at 0°, and R_1 its resistance at 100°, we may establish for the particular wire a temperature scale, which we may call the *scale of platinum temperatures*, such that if R be the resistance at any temperature T° on the air-scale, this temperature on the platinum scale will be $\frac{R - R_0}{R_1 - R_0} \times 100°$. For this quantity, CALLENDAR employs the symbol *pt*, its value depending on the sample of platinum chosen.

In order to reduce to the standard scale of temperature the indications of any platinum thermometer, it is necessary to know the law connecting T and *pt*. These are, of course, identical at 0° and 100°, but the determination of the remainder of the curve expressing the relationship between them is a matter for experiment. Our present knowledge of this relation depends mainly on the investigations of CALLENDAR and GRIFFITHS.

The following is a list of the principal papers published bearing on the subject of platinum thermometry :—

CALLENDAR	'Phil. Trans.,' A, 1887.
,,	'Phil. Mag.,' July, 1891.
,,	,, July, 1892.
,,	,, February, 1899.
GRIFFITHS	'Brit. Assn. Reports,' 1890.
,,	'Phil. Trans.,' A, 1891.
,,	,, 1893.
,,	'Proc. Roy. Soc.,' vol. 55, 1894.
,,	'Nature,' November, 1895.
,,	,, February, 1896.
CALLENDAR and GRIFFITHS	'Phil. Trans.,' A, 1891.
CAREY FOSTER	'Nature,' August, 1894.
HEYCOCK and NEVILLE	'Chem. Soc. Journ.,' 1890.
,, ,,	,, ,, 1895.
GRIFFITHS and CLARK	'Phil. Mag.,' December, 1892.
CLARK	'Electrician,' vol. 38.
OLSZEWSKI	'Phil. Mag.,' August, 1895.
DEWAR and FLEMING	,, September, 1893.
,, ,,	,, July, 1895.
HAMILTON DICKSON	,, December, 1897.
,,	,, June, 1898.
D. K. MORRIS	,, September, 1897.
WAIDNER and MALLORY	,, August, 1897.
DAY	,, August, 1897.
CHREE	'Nature,' July, 1898.

The work of CALLENDAR established for a particular sample of pure platinum the relation

$$d \equiv \mathrm{T} - pt = \delta \left[\left(\frac{\mathrm{T}}{100}\right)^2 - \frac{\mathrm{T}}{100}\right]$$

over the range 0° to 600°. For CALLENDAR's wire the value of δ was about 1·57.

Subsequent experiments by CALLENDAR and GRIFFITHS showed that the values of δ for the different samples of platinum they examined varied greatly with their purity, yet, provided that the percentage of impurity were small, the formula given above held true. They found from their experiments that the $T - pt$ curve was always a parabola, and that, therefore, to establish the whole curve showing the divergence of the two scales, it was sufficient to know d for three fixed points. For two of these, viz., 0° and 100°, d is by definition zero. For the third point, for reasons indicated in their paper, GRIFFITHS and CALLENDAR chose the boiling-point of sulphur, and subsequently made a new determination of this point by an air thermometer, finding as their most probable value 444°·53, the pressure being 760 millims. This value, which is nearly four degrees lower than that previously obtained by REGNAULT, is the one which has been generally adopted in work with the platinum thermometer.

As further evidence in confirmation of this conclusion GRIFFITHS points out that, if this number be taken for the boiling-point of sulphur in the calculation of the values obtained by him for the boiling and freezing points of a number of substances on which he experimented, the results for most of the substances concord better with their accepted values as determined by other observers, than if REGNAULT'S value, 448°·34, be adopted.

Many of these accepted numbers quoted in GRIFFITHS' paper are given to hundredths of a degree, but closer examination of the original papers in most cases reveals the fact that the reductions to the normal scale and the various corrections of the thermometers employed, if made at all, are, to say the least, very uncertain. Further, we see no *a priori* reason why, in GRIFFITHS' experiments, the results with certain of the substances employed should be rejected from consideration, as there does not appear sufficient ground for supposing that the experimental error in these cases was higher than the average.

Substantially then our knowledge of temperatures, deduced by means of the platinum thermometer, depends solely on the correctness of the conclusions of GRIFFITHS and CALLENDAR :—

(1) That the boiling-point of sulphur under 760 millims. pressure is 444°·53.

(2) That the curve representing the divergence of the platinum and air scales is a parabola.

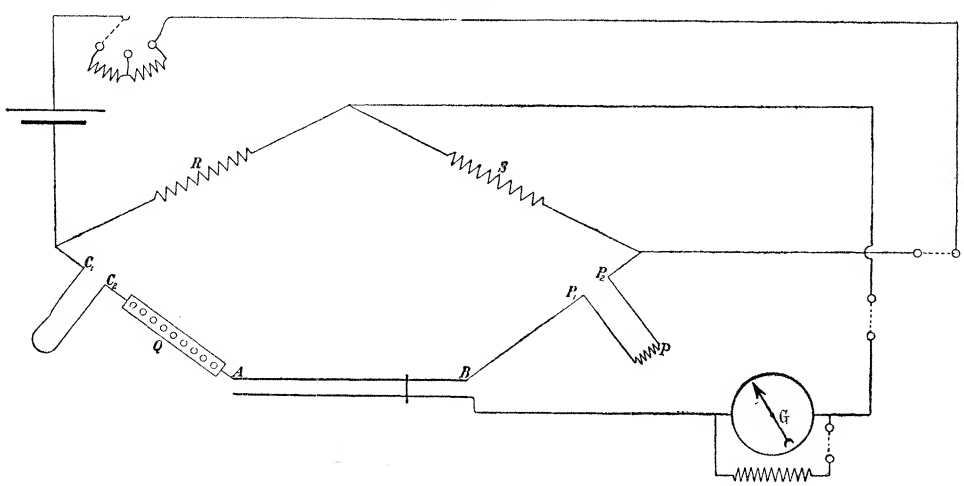
II. THE INVESTIGATION FOR THE KEW COMMITTEE.

In recent years the platinum thermometer has been employed by various observers, and their experience has tended to confirm the view that it could be relied upon to give constant indications at a given temperature. It consequently appeared to the Kew Observatory Committee that it might be possible to use this instrument as a means of referring measurements of temperature to the scale of the gas thermometer adopted as an International standard by the Comité International des Poids et Mesures, and thus to extend their means of accurately testing thermometers sent to them for verification at temperatures outside the range 0° to 100°. With this view they deemed it desirable to obtain an independent investigation into the principles and methods of platinum thermometry, and they consequently procured a complete equipment of the necessary apparatus, which was installed at the observatory under the supervision of Mr. GRIFFITHS in a special building. As the general results of the experiments made with this apparatus seemed promising, the Kew Committee approached the Comité International des Poids et Mesures, with a view to securing their co-operation, and ultimately it was arranged that a direct comparison, extending over as wide a range as possible, should be made between some platinum thermometers belonging to Kew and the Standard instruments at the International Laboratory at the Pavillon de Breteuil, at Sèvres, near Paris. The present paper is the outcome of this investigation, in which it may be understood that one of us (C.) is responsible for the gas and mercury thermometry involved, while the working of the platinum thermometers devolved on the other (H.). In it will also be found an account of the means by which the range of the gas thermometer employed was extended upwards from 200°, the limit of the Bureau's previous experiments, for the purpose of this investigation.

III. THE FIRST FORM OF PLATINUM THERMOMETER RESISTANCE-BRIDGE.

As a full account of the first platinum thermometer apparatus acquired by Kew Observatory has been published by GRIFFITHS ('Nature,' Nov. 14, 1895), under whose supervision it was standardized, it is unnecessary here to give more than a general description of its chief features. A diagrammatic representation of the connections is given in fig. 1.

Fig. 1.



Q	Resistances of bridge.	C	Compensator of thermometer.
R and S	Proportional coils.	AB	Bridge-wire.
P	Thermometer spiral.	G	Galvanometer.

Here R and S represent the proportional coils of about five ohms each, adjusted to exact equality. P is the thermometer coil connected by two long flexible copper ends to the box terminals P_1 and P_2.

The wires in the stem of the thermometer leading to the coil are of thick platinum, the coil itself being of a very pure sample of platinum ·006 inch in diameter. Down the stem run also a second pair of leads made as similar as possible to the coil leads, but connected together at their lower extremities and having no contact with the coil. This loop, connected at C_1C_2 in the figure in the opposite arm of the bridge to the thermometer itself, serves to compensate the changes in resistance of the thermometer leads proper, due to variations of stem temperature. The four copper wires joining P_1P_2, C_1C_2 to the thermometer are plaited together into a single cable, so that temperature changes throughout their length may affect them all equally.

Q represents the nine platinum-silver resistances of the box connected to one another in series, the lowest coil having a resistance of 5 box-units (1 box-unit = ·01 ohm), and the rest forming a series 10, 20, 40, 80, up to 640 units the largest coil. An extra coil of 100 units is used to determine the fundamental interval of the usual type of thermometer, whose change of resistance between 0° and 100° is 100 box-units, *i.e.*, 1 ohm.

A platinum silver bridge-wire, AB, provided with a scale of millimetres, furnishes the means of balancing exactly any resistance of P. A special form of slider is employed for the contact between the bridge-wire and a precisely similar wire stretched parallel to it, connected to the galvanometer. The exact position of the transverse wire forming the contact-piece is indicated by a vernier by which ·01 millim. may be estimated. This symmetrical arrangement of two similar wires is found to diminish the thermoelectric effects at the movable contact.

Coils of 20 and 100 ohms are provided as resistances in the battery circuit, and also a " tenth " shunt for the galvanometer.

The top of the resistance-box is of a special quality of marble of good insulating properties.

The whole is enclosed in a double-walled tank holding a considerable mass of water, which is kept at a constant temperature near 20°, by a regulator controlling a small gas flame. A delicate thermometer suspended in air in the interior of the box indicates the coil-temperature, and the whole of the upper surface of the box is protected against radiation and air currents by a glass cover similar to a balance case.

IV. Experiments with the First Apparatus.

From the time of the acquisition of this apparatus determinations were repeatedly made of the constants of each of the platinum thermometers, in order to test the permanence of the whole arrangement under ordinary working conditions; also to ascertain how the accuracy obtained was influenced by alterations in the external conditions of experiment, such as changes in the laboratory-temperature, the different treatment of the apparatus by different observers, &c.

These trials, which were continued over a considerable period, showed that one of the disadvantages of this form of apparatus is the almost inevitable difference of "lag" between the mercury thermometer employed to indicate the coil-temperature, and the platinum-silver coils themselves, which in this case hang loosely in air. From this cause, especially when the box-temperature is changing rapidly, some uncertainty as to the coil-temperatures is introduced. During the winter, when the temperature of the laboratory often fell very considerably during the night, and also in summer when it rose to over 20°, the temperature of the *coil-space* changed rapidly during the daytime, although the regulator nevertheless maintained the water in the tank very near 20° throughout, showing that the protection afforded by the glass cover of the resistance-box was insufficient under the prevailing conditions. The measurements made showed conclusively that in this case the coils followed temperature changes *faster* than the mercury thermometer selected to indicate the coil-temperature.

The temperature-coefficient of the alloy of which the coils are constructed is ·00026 and that of the platinum wire of the thermometers is ·00386; if then we wish to determine a platinum-temperature to ·001° (whatever the resistance of the thermometer chosen) we must know the coil-temperature to ·015°. Therefore, unless great precautions be taken with the mercury thermometer, it is difficult to see how the measurements of coil-temperature can be sufficiently trustworthy.

Griffiths in his later experience has got rid of the first-mentioned difficulty while retaining platinum-silver as the resistance metal, by immersing the coils in a well-stirred bath of highly insulating oil, into which the mercury thermometer is placed directly, thus rendering the measurement of the coil-temperature much more certain.*

V. Construction of the New Apparatus.

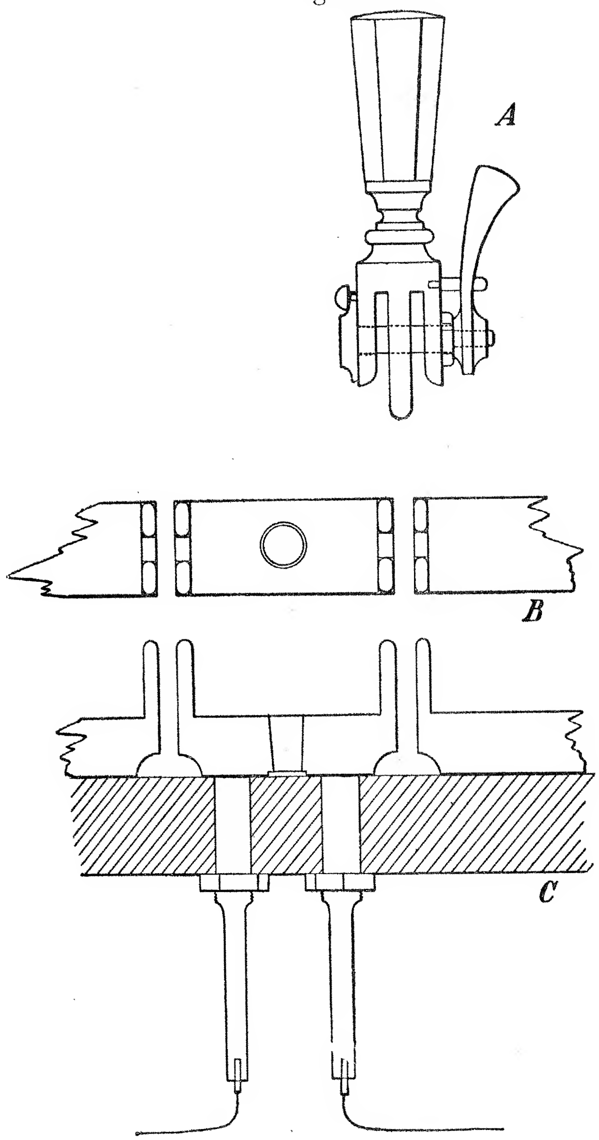
As it was anticipated that the experiments at Sèvres might occupy some time, and it was not thought advisable that the Observatory should be deprived altogether of the use of platinum thermometers for a long period by this apparatus being taken to France, a new resistance-box was ordered specially for this work. The construction of this box was entrusted by the Committee to Messrs. Crompton and Co., Limited, and its behaviour has on the whole been very satisfactory.

In view of the fact that it was not easy to maintain the platinum-silver coils at a sufficiently uniform temperature winter and summer by any simple means, and in view of the difficulty previously mentioned as to the indication of the true coil-temperature with sufficient accuracy by a mercurial thermometer, it was decided in

* A description of Griffiths' subsequent improvements on the original Kew apparatus, here described, is given by G. M. Clark ('Electrician,' vol. 38, p. 747).

this second apparatus to obviate the necessity of very accurate measurement of the coil-temperature by using one of the new alloys of very small temperature-coefficient, manganine being the one chosen. The expediency of this change was subsequently emphasised by the fact that we found it was inadvisable to artificially heat the room at Breteuil in which the comparisons were made, on account of the uncertainties

Fig. 2.



attending the measurement of the temperatures of the various mercury columns of the gas thermometer. During the year and a-half the experiments lasted, the room temperature varied from about 4° to 23°, which would have rendered accurate artificial control of the box-temperature extremely difficult.

Since in the first resistance-box the thermoelectric effects between the various wires and terminals in the circuit (in which several different metals are used) were sometimes considerable, copper was substituted throughout for brass in the new box, the only metals in circuit being copper and manganine. For the platinum-silver bridge-wire was substituted a manganine strip heavily gilt, placed on edge and stretched between two adjustable clips. The slider is provided with a fine adjustment, which can be manipulated *from the outside of the box*, without risk of heating the galvanometer-contact by repeatedly approaching it with the hand. As in Mr. GRIFFITHS' latest form, the terminals project outside the glass case. The top of the box is formed of a heavy slab of white marble 75 centims. long, 30 centims. wide, and 3 centims. thick. For the ordinary form of plug-contacts are substituted heavily gilt forks of forged copper, which can be clamped by powerful steel screws over tongues projecting upwards from the blocks to which the coils are fastened. A general plan of the resistance-box is shown in Plate 1 and the details of one of the contacts in fig. 2.

VI. THE RESISTANCE-COILS.

The general scheme of the box connections is almost the same as the one previously described, and may be traced in fig. 1. For the winding, fixing, and annealing of the manganine coils the method described in the official publication of the Physikalisch-Technische Reichsanstalt at Charlottenburg was carefully followed. The specimen of wire used was selected after various tests from several furnished by Messrs. W. T. GLOVER and Co., of Salford, and was double silk covered No. 26 S.W.G. The diameter of several pieces cut from different parts of the bobbin only varied within very narrow limits. In order to simplify the application of the temperature correction, the same wire was employed for all the coils except the two lowest. These were of strip manganine, and being originally cut off too wide, could be adjusted till accurate by clipping the edge with shears.

VII. COIL VALUES ADOPTED.

In the Callendar-Griffiths resistance-boxes the coils are arranged on the binary scale, and the value of each is determined in terms of the sum of those below it together with a certain length of bridge-wire. Although Mr. GRIFFITHS gives evidence for the utility of this arrangement in general work, it was thought more important, for the purposes of this research, to have several independent checks in the determination of each coil-value, than that the maximum resistance, measurable with a given number of box-coils, should be as high as possible.

The thirteen coils were therefore arranged as follows, the values being expressed in ohms :—

40		20		10		4		3		2		1
A		B		C		D		E		F		G
	·02		·05		·1		·2		·3		·4	
	N		M		L		K		J		H	

Several of these values are the ordinary ones adopted for standard resistances, so that with a suitable arrangement comparisons with a standard could be made from time to time. Coils of these values are also much easier to measure off accurately than the larger multiples of ·01 ohm on the binary scale. We will revert to this question as to the best arrangement of coil-values in the discussion of the method of standardization adopted.

The resistance box is fitted with three interchangeable bridge-strips of different resistances, on which a change of ·01 ohm in the box-coils causes a displacement of the slider, to restore the balance, of 10, 5, and 2 centims. respectively. The slider is fitted with a vernier giving $\frac{1}{20}$th millim. directly, it being considered unnecessary, perhaps indeed impossible, to determine the position of the knife edge forming the contact to a greater accuracy than this, without taking extraordinary precautions with the scale, the slider, and the bed in which it travels.

With the three bridge-strips the resistance corresponding to a movement of one millim. is ·0001, ·0002, ·0005 ohm, respectively.

The strip of medium resistance was the one employed exclusively during this research.

The slider is of the form employed on the best potentiometers, and can be displaced either by hand or by a fine-adjustment screw with large milled head, projecting beyond the outside of the case. This screw moves longitudinally a rectangular frame carrying the slider; this frame is also capable of lateral movement in the massive brass casting which surrounds the bridge-wire, and tends to protect it from injury, and to equalise its temperature from end to end. The return contact from the slider to the galvanometer was originally made by means of this movable frame, but from some unexplained cause, apparently not thermoelectric, this led to unsteadiness of the galvanometer zero.* Coupling the various parts of the framework together electrically by flexible copper wires did not remove the difficulty, and ultimately it was found best to have a silk-covered return lead attached to the spring contact on the slider, and to cut off the frame from all electrical connection with the apparatus.

The marble slab forming the top of the resistance-box was supported from the inside of the tank by an iron framework, carrying racks for the coils, the sides being left quite open, and all the coils easily accessible for inspection at any time. The whole was placed in a very heavy double-walled copper trough holding about 50 litres of water, and was covered by a doubly-hinged lid, glazed with thick bevelled plate

* In these experiments, in which a Griffiths' thermoelectric key (described later) is used, in the normal position of the key the galvanometer circuit remains *made*. When the platinum thermometer is not changing rapidly in temperature, the stability of the galvanometer zero is a good criterion, from which much may be gathered as to the working state of the bridge, and the magnitude of the thermo-currents present. We have reason to believe from our own experience that the use of a well-constructed key of this type considerably facilitates the carrying out of low-resistance measurements, where high accuracy is desired.

glass. Provision was made in the outer space of the tank for a regulator and heating arrangement underneath. This was, however, not used during the present experiments. Suitable thermometers indicated the temperature of the water in the outer tank, and two sensitive ones, divided to tenths of a degree with thin bulbs and their stems bent at right angles, indicated the temperature of the internal coil-space.

IX. Galvanometer Shunts and Battery Resistances.

The resistance-box was provided, as in the Callendar-Griffiths type, with a set of galvanometer shunts and a series of battery resistances of 20, 50, 100, and 500 ohms. It was afterwards found a great advantage to have a more exact adjustment of the battery current, and for this purpose a subsidiary three-dial box, working up to 1,000 ohms, was provided.

Previous experience at Kew had shown the occurrence of differences in the point of balance, according as the battery-current was in one direction or the other; it was found that the difference between the readings with the current in the two directions generally increased gradually during the first quarter of an hour on commencing the observations, and was greater the greater the intensity of the battery current. In order to be able to study, and if possible to eliminate this cause of uncertainty, we placed in the battery circuit of the new apparatus a high-insulation reversing switch. The working of this switch was at first unsatisfactory, but was subsequently perfected by short-circuiting the rubbing contacts at the pivots by flexible brass strips, and covering the five contact studs with thin platinum plates.

X. Battery Power Employed.

The battery used throughout the experiments consisted of two dry-cells of the Obach type, obtained from Messrs. Siemens; we ascertained that the E.M.F. of the two cells was practically constant throughout and about 2·8 volts, and that the internal resistance of the two in series did not rise to more than 1 ohm, changing by a quantity quite negligible in comparison with the large resistance always added in the battery circuit.

XI. Thermoelectric Key.

For the completion of the different circuits a Griffiths' thermoelectric key was employed, as in the first Kew apparatus. The essential feature of this key consists in the addition to the ordinary form of double bridge-key of a lever so arranged that when the key is released the galvanometer circuit remains made. Thus a simple depression of the key first breaks the circuit of the galvanometer, then makes that of the battery, and finally remakes the circuit of the galvanometer.

The key we used was somewhat modified from the original pattern, which, being mounted on wood, was not found to be quite perfect as regards insulation. The new key, which along with the thermometers and nearly all the accessory apparatus was obtained from the Cambridge Scientific Instrument Company, Limited, is shown in fig. 3, and in the general plan of the auxiliary apparatus (fig. 4). It is provided with

Fig. 3.

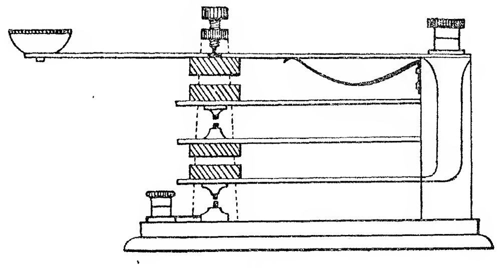
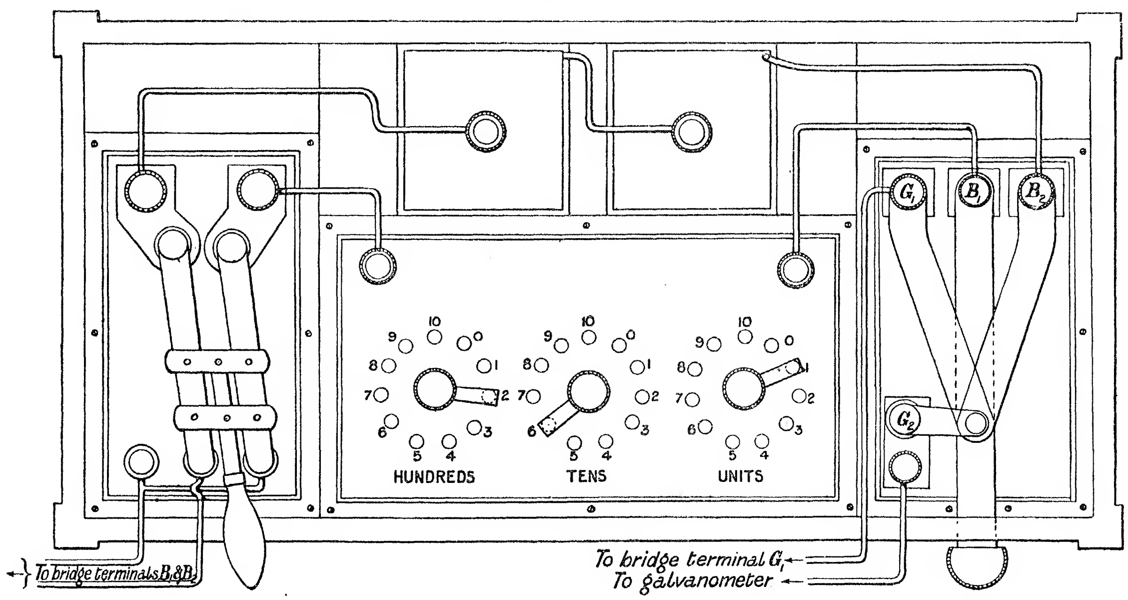


Fig. 4.



ebonite pillar insulation, and the four levers are rearranged in their order, the two forming the galvanometer-contacts being supported from the same pillar one under the other. All the contacts in the key are of platinum. An adjustable steel spring under the topmost lever helps to hold it up against the contact screw above, thus ensuring good contact in the galvanometer-circuit when the key is released.

XII. Accessories to the Resistance-Bridge.

The battery, reversing-switch, key, and external resistances were all enclosed in a wooden case provided with a glass lid, the necessary handles for the adjustments and for working the keys, projecting through the sides, and the whole being kept nearly air-tight by suitable protections. This seemingly unimportant detail we found to be a great advantage, as in the damp weather, experienced during part of the experiments, the insulation of exposed parts always required considerable attention. The battery was insulated from the wooden case by gutta-percha strip, and after this was added, the insulation resistance of the whole apparatus, when all was kept dry, was practically perfect.

XIII. Galvanometer.

For the first experiments the galvanometer employed was one of the pattern described by Dubois and Rubens in 'Wied. Annalen,' vol. 48, p. 236, lent to us by Professor Schuster. This is a Thomson four-coil instrument with connections so arranged that its bobbins may be coupled to give an internal resistance of 80, 20, or 5 ohms. The magnet system and mirror weighed together 0·2 gram. The deflections were observed from about three metres distance by a large Steinheil telescope. Much trouble was experienced in finding a foundation for the galvanometer sufficiently free from vibration. After several unsuccessful experiments in which we attempted to insulate the galvanometer with rubber blocks, a special pillar was erected independent of the floor. We found, however, that, even when resting on this, the vibration of the magnet-system, caused by heavy traffic on the Versailles road, was sufficient at intervals to prevent any satisfactory observations being made. At this juncture Professor Carey Foster was appealed to, and through him Mr. R. K. Gray, of the India-rubber, Gutta-percha, and Telegraph Works Company, of Silvertown, very kindly came to our aid by sending us a reproduction of an arrangement he had employed at the works to cut off vibrations from delicate instruments. It consisted of a brass plate forming a platform from which the galvanometer is suspended, the whole being slung by long india-rubber tubes from a wall-bracket above. To diminish the effect of air currents we added a damping arrangement consisting of a vertical metal cross with attached horizontal vanes, plunging into a vessel standing on the concrete pillar and containing a thick oil. The galvanometer and suspension were also completely surrounded by a paper screen extending upwards to the ceiling, provided with suitable openings for making the adjustments.

The india-rubber suspension arrangement, when once the tubes were properly stretched, worked perfectly satisfactorily till the winter, when, presumably under the influence of the low temperature of about 4° or 5°, such a change took place in the elasticity that we were obliged to seek a substitute for the india-rubber, less influenced by climatic conditions.

We at length managed to construct from steel wire, 1 millim. in diameter, long spiral springs of the requisite strength, which have served the purpose admirably, and at the same time have shown a comparatively small variation of elasticity with temperature. The arrangement of the suspension in its modified form is shown in figs. 5 and 6.

Fig. 5.

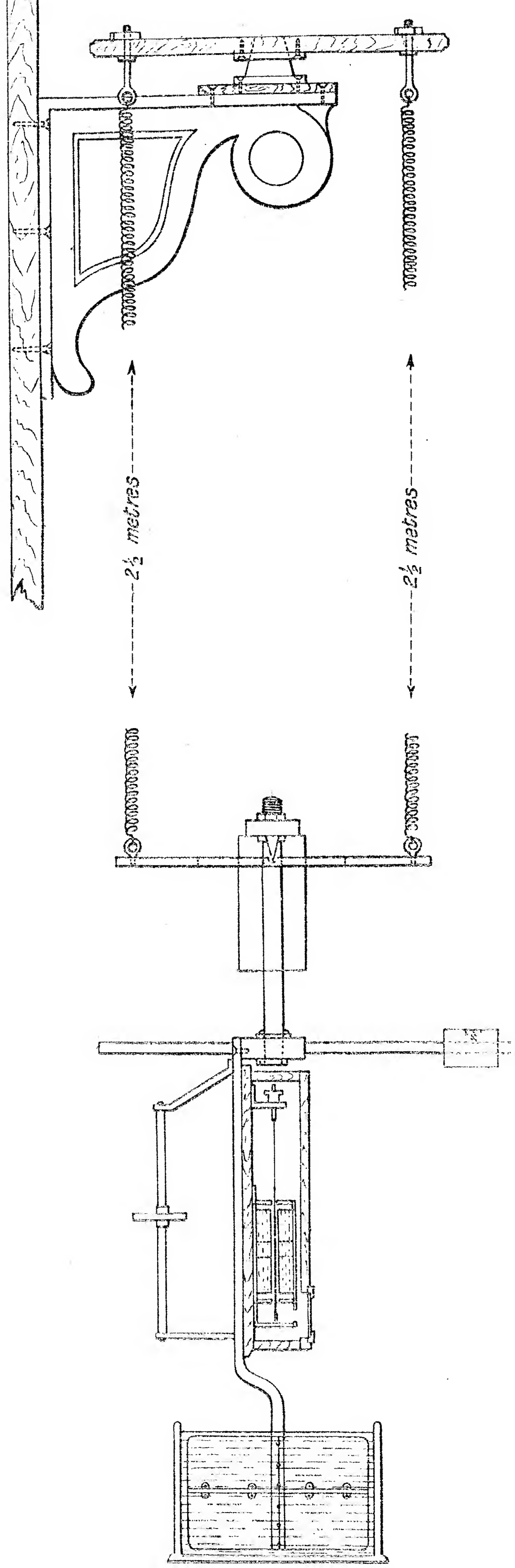
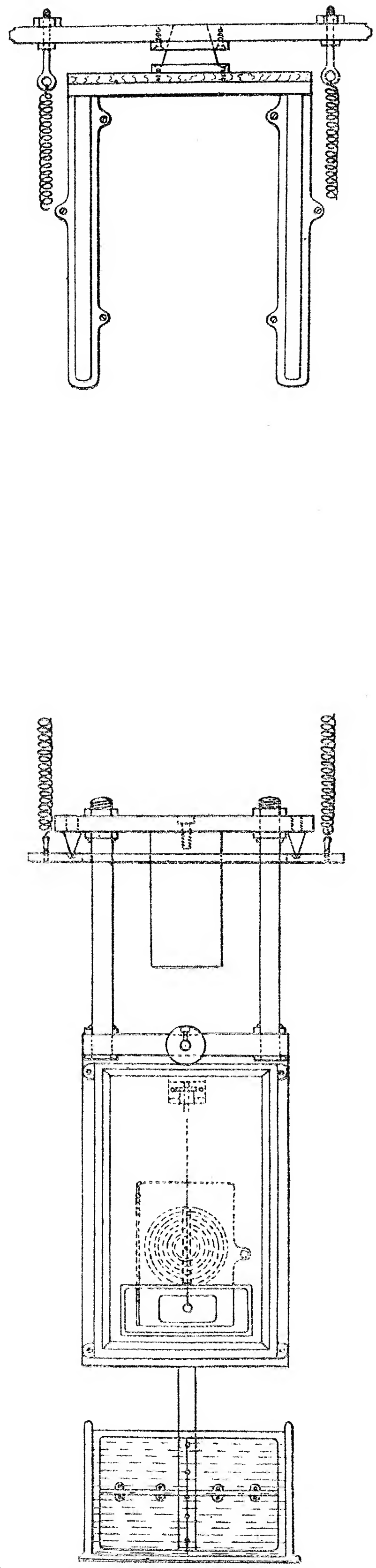


Fig. 6.



With the comparatively heavy magnet-system of this first galvanometer, we were obliged, in order to obtain the requisite sensibility, to work with a relatively long time of vibration, which was not convenient for rapid work, and it was ultimately found necessary to so modify our galvanometer that the necessary sensitiveness should be obtained with a time of swing not exceeding six or seven seconds. We therefore had recourse to the ingenious type of magnet-system devised by M. BROCA, of the Ecole de Médecine, Paris, and described by him in the 'Journal de Physique,' February, 1897. In its latest form this consists of two vertical hollow magnets, having at their middle points a north and south pole respectively. When the two similar but oppositely magnetised needles are fixed strictly parallel to one another, the system thus formed is perfectly astatic in a uniform field, since the strength of the pole at the centre of each magnet is equal to the sum of the poles at its ends, and further the astaticism is not affected by even complete loss of magnetism in one of the magnets. M. BROCA was kind enough to lend us a galvanometer of the type described in his paper, and a magnet-system made by himself. This instrument is the one shown in the sketches of the galvanometer and suspension in figs. 5 and 6. We had not the means of accurately measuring the sensitiveness of this instrument by one of the ordinary processes, but ascertained that, with a time of swing of five seconds, the scale deflection, for a want of balance of the bridge corresponding to ·001°, was about 0·5 millim. This was with a system carrying a mirror large enough to give a bright image in the telescope, readable without difficulty in broad daylight.

XIV. THERMOMETER LEADS.

The thermometer leads were of stranded copper equal to No. 17 S.W.G., and about seven metres long. The resistance of the four separate wires was carefully equalised before fixing on the copper end-pieces used to make the contacts, each loop, P_1P_2, C_1C_2, having a resistance of ·15 ohm at 16°. After nearly two years continual use the two loops differed in resistance by ·0003 ohm, a change of only about one-fifth per cent. of the whole.

The connections between the resistance-box leads and thermometer were made by means of the special alloy employed by Mr. GRIFFITHS, and from our own experience we can strongly recommend these joints as very trustworthy and easy to make.

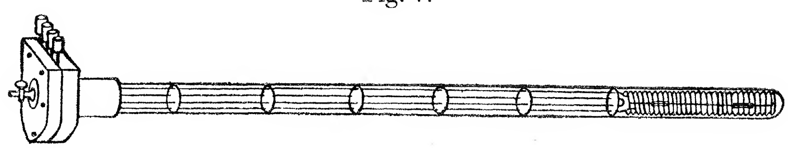
XV. THE PLATINUM THERMOMETERS.

The resistance of all except one of the platinum thermometers belonging to the original Kew installation was such that their change of resistance between 0° and 100° was almost exactly one ohm. Though convenient for high range work, this type of thermometer is hardly suitable for measurements of the highest accuracy at lower temperatures, in consequence of the relatively considerable effect on the results of small variations in the plug-contact resistances. On this account two new thermometers of higher resistance were ordered from the Cambridge Instrument Co.; their fundamental intervals were five and ten ohms respectively. They are designated in this paper K. 8 and K. 9.

Owing to the shape of the various baths in which the comparisons with the platinum thermometers were made, and more especially to the difficulty of keeping dry the air within the tubes of thermometers of the old form, we were obliged to modify the form of the "head" of these principal thermometers.

In the reconstruction the whole thermometer was arranged so as to be practically air-tight, and the contacts were rearranged in such a manner that although the four wires all left the thermometer at the same side, yet the "coil" and "compensator" arms were perfectly symmetrical. At the same time the number of contacts where thermoelectric effects could arise was reduced as far as possible, by suppressing the brass terminals and making the connection between the platinum leads and the copper fusible metal cups *directly* by stout copper wires, all joints being made quite secure with hard solder. Though this form of thermometer-head is a little more difficult to construct, we find when the four contact cups are surrounded by a thin shield of polished metal to keep off air-currents, that the thermo-effects, almost invariably present to *some* extent in the old form of thermometer, especially when rapid temperature changes are progressing, are almost entirely absent. Another feature is the readiness with which the glass tube can be freed from internal moisture by simply

Fig. 7.



connecting the small stopcock on the ebonite plate alternately to a vacuum pump and to an arrangement for supplying dry air, while the whole thermometer is at a high temperature. This we find to be of great importance for accurate work below 100°.

A sketch of the thermometer in its improved form is given in fig. 7.

XVI. Standardization of the Resistance-Box.

The standardization of the resistance-box consisted in the determination :—

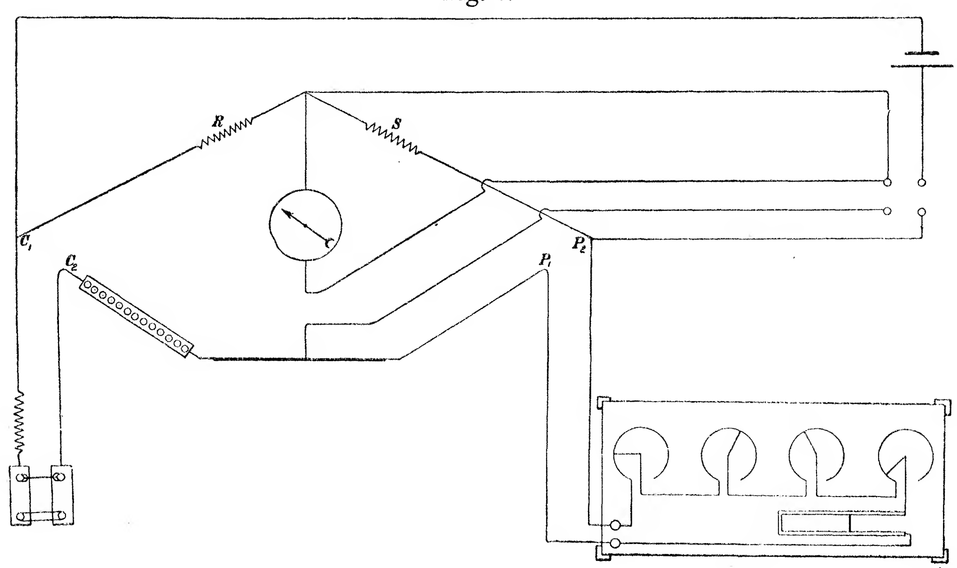
(1) Of the calibration corrections of the bridge-wire;
(2) Of the values of the resistance coils in terms of one another;
(3) Of the temperature-coefficient of the coils.

XVII. *Calibration of the Bridge-Wire.*

As has previously been mentioned, the cylindrical bridge-wire employed in the

CALLENDAR-GRIFFITHS box was replaced by a manganine strip, cut from a large sheet. Although this strip had been carefully adjusted by filing to a very fairly uniform resistance along its whole length, yet, from the method of its construction, we anticipated the possibility of there being in some places more sudden variations of resistance than were likely to occur in a wire of a hard material like platinum-silver or platinum-iridium carefully drawn down to a certain diameter. We determined therefore to substitute for the usual GAY-LUSSAC calibration a more complete one with several different "columns." As it was not always possible to take vernier-readings with the slider close up to the ends, we decided to employ only the middle 48 centims., and to obtain the calibration corrections for each 2 centims. over this range. We are indebted to Dr. BENOIT for his advice on the best method to adopt. He recommended that the whole length should be divided into two parts, and that for each part a "complete" calibration should be made for every 4 centims., involving the employment of "columns" of 20, 16, 12, 8, and 4 centims., and that afterwards the intermediate 2 centim. points should be determined by subdivision of each interval of 4 centims. into two parts. The necessary conversions of the separate corrections found to one system were made exactly as in the calibration of a mercury thermometer or a divided scale.

Fig. 8.



The method adopted for making the necessary measurements is described in a paper presented to the Royal Society in 1896 by one of us; the scheme of the connections is shown in fig. 8.

Between the terminals P_1P_2 of the resistance-box is connected an auxiliary adjustable resistance, having four sets of coils made of the same sample of manganine as those of the bridge proper, and also a small U-shaped trough containing mercury. By means of this appliance, a plan of which is shown in fig. 9, the resistance between

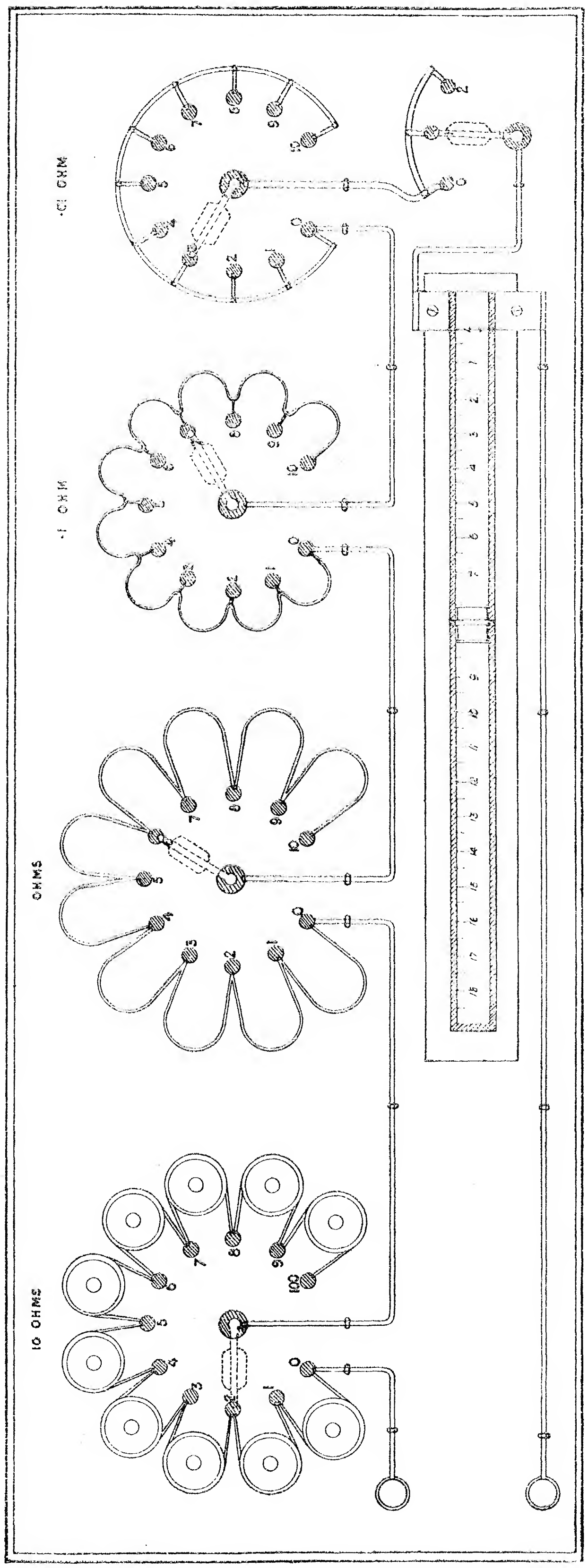


Fig. 9.

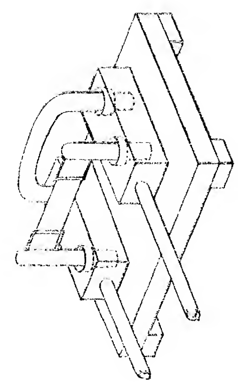


Fig. 10.

the terminals P_1P_2 can be quickly adjusted to any value between zero and 100 ohms. In the opposite arm of the bridge, between the terminals C_1C_2, are inserted a fixed resistance of 0·1 ohm and the calibrating apparatus. This consists of two massive copper blocks of rectangular form, mounted on an insulating base, each pierced by two holes about a centimetre in diameter, which are well amalgamated and partly filled with mercury. Into one pair of holes are inserted two round copper pillars, across which is soldered a piece of thin manganine strip, and into the other pair of holes the lower ends of a thick U-shaped copper rod. A sketch of the calibrator is given in fig. 10.

A number of strips of different resistances, each mounted between copper pillars as shown, are first prepared, the values chosen being equivalent to 2, 4, 8, 12, 16, 20, and 24 centims. movement of the slider. The calibration is commenced by placing the contact-maker to the division 24 to the left, and one of the strips in position on the calibrator. The resistance in the opposite arm of the bridge is then adjusted so that no galvanometer deflection is obtained, and the exact position of the slider noted. The manganine strip remaining untouched, the copper short-circuiting piece is now placed across between the two remaining mercury cups, and the slider is then moved to the right till the balance is again restored. In order to eliminate the effects of any gradual temperature changes, the process is repeated, the readings being made in the reverse order, a similar pair of readings being made for each successive position along the bridge-wire. The results of several series of observations made on different days with each strip are then combined, and the whole set treated precisely as an ordinary calibration of a length or volume, and the curve of corrections prepared.

It was interesting to compare the results of the complete calibration with those deduced from the observations with the two-centim. column alone, and from two test calibrations made by means of the coils M and N. The general agreement of the different results was found to be satisfactory.

XVIII. *The Resistance Coils.*

The manganine coils were annealed, in accordance with the recommendations of the German Reichsanstalt, by heating them to about 140° for some time and allowing them to cool slowly. This was done in a closed electrically heated space in which the temperature could be regulated at will, and the cooling could be made as slow as desired.

The ends of each coil were hard-soldered to copper tags of rectangular form previous to the final annealing, these tags being afterwards firmly fastened by ordinary solder to the stout tinned copper leads connected to the contact-blocks.

The coils were wound on glass tubes 3 centims. in diameter, which were fastened by metal strips to wooden cross-pieces supported from the iron framework of the resistance-box. These tubes were coated with three thin layers of shellac varnish before the coils were wound, and after the winding the wire itself was also well varnished to improve the insulation and to protect it from oxidation during the annealing.

We understand that the standard manganine coils issued by the Reichsanstalt are not tested till a year after their construction, but that after this lapse of time the gradual changes they exhibit are very small.

In our case we were however obliged to begin work with the resistance-box before the coils had been properly aged, and therefore were not surprised at alterations in their values, particularly during the first few months.

We regret that at the commencement of our work we had not at our disposal the means of comparing the coils with an invariable standard, but could only obtain their relative values in terms of the mean bridge-wire unit, which was even more likely to change slightly than the coils themselves; on this account we are unable to give details of the progressive alterations in their absolute values, and can only indicate the means we adopted to prevent these changes influencing the accuracy of our temperature-measurements. The changes were, as was to be expected, most appreciable at the beginning of the work. The first standardization was made as soon as the apparatus was got into order and fitted up at Breteuil, and immediately following this came the comparisons of K. 8 with the mercury thermometers. As soon as this series of comparisons was completed a second standardization was at once made. The individual observations of the thermometric fixed points and comparisons were then reduced with both the earlier and new coil-values. At a later stage it was found that, although the absolute value of the mean bridge-wire unit had slightly altered, yet the values of the box coils relatively to one another, with the exception of one of the very low resistances constructed of strip manganine, had not changed by an amount large enough to make the two determinations differ materially. It was easy to allow for such small variations as did occur by taking into consideration the date of each experiment and assuming that the change between the two standardizations was proportional to the time which had elapsed since the first.

Not counting a preliminary series of observations, four complete standardizations were made in all during the course of the work with the thermometers, and we think that no serious errors were introduced into the results by the alterations in the relative values of the resistance-coils.

Further particulars as to the changes in the values of the different coils are given on p. 58 after the description of the method adopted for the standardization.

For the comparison of the coil-values with one another the following plan was adopted. Firstly, the values of the smaller coils M and N were determined directly in terms of the bridge-wire by the same process as was employed in the calibration. Next, each higher coil in turn was balanced against the adjustable manganine resistance previously described, which in each case was so adjusted that the position of the contact-maker on the bridge-wire was in the neighbourhood of the zero of the scale. The coil in question was then changed for some combination of those of lower values giving the same (or very nearly the same) nominal resistance, and the outside resistance remaining untouched, the contact-maker was again adjusted to equilibrium.

The distance between the first and second positions of the slider is a measure of the difference between the two sets of coils, expressed in mean bridge-wire divisions. The process being repeated for all the coils and the different combinations equivalent to each, the results are collected into a set of equations of the following form* :—

$$\begin{array}{lr} A - B - C - D - E - F - G & = a_1 \\ \quad \&c. & \\ D - E \quad - G & = a_4 \\ D - E \quad - H - J - K - L & = a_5 \\ & \&c., \&c. \end{array}$$

As previously mentioned, the scheme of coil-values adopted was such as to permit of independent values for most of the coils being obtained in a single standardization, the difference between these several values being a measure of the accuracy obtained.

For the standardization we adopted the same scheme in the four sets of determinations of the coil-values taken at different times throughout the research. We ascertained during the experiments, but too late to make any change, that the values chosen for the higher coils were not such as were best adapted for giving a number of inter-dependent relations, and on this account the control only extended upwards to the fourth largest coil.

We give below the residual errors obtained in one of the standardizations by substitution of the values found by least squares for each coil in the respective equations of condition, suppressing the first three coils for which the control was absent.†

* The method of forming these equations will be readily seen on reference to the table of coil-values given previously on p. 45.

† In the opinion of Dr. BENOIT, whose kindness in giving us his advice with regard to the methods of standardization we here gratefully acknowledge, the best way to obtain in one standardization the requisite number of equations from which the relative values of such a system of coils can be satisfactorily determined, is to adopt a system similar to that employed for standard sets of weights. After careful consideration of these we think the following scheme for a set of fifteen coils would be almost an ideal one. Without counting combinations only involving changes in coils, whose resistance is small compared to the total in any comparison, we should have in this system several controls for each coil-value

$$\begin{array}{ccccccccccccccc} 20' & & 20'' & & 10' & & 10'' & & 5 & & 2' & & 2'' & & 1' \\ & \cdot05'' & & \cdot05' & & \cdot1 & & \cdot2'' & & \cdot2' & & \cdot5 & & 1'' & \end{array}$$

The system of about 44 equations of condition to determine the unknowns, given by the different direct comparisons, can either be divided into groups and solved thus, or may be solved as a whole, which, if GAUSS' method be followed, can be done without undue labour, as the coefficients of the various terms remain small whole numbers till near the end of the resolution.

	Residual error in bridge-wire units.
D − E − H − J − K − L = n_1	− 0·0158
D − F − G − H − J − K − L = n_2	+ 0·0022
D − E − G = n_3	+ 0·0136
E − F − G = n_4	+ 0·0087
F − G − H − J − K − L = n_5	+ 0·0000
G − H − J − K − M = n_6	+ 0·0071
H − J − L = n_7	+ 0·0024
H − K − L − M − N = n_8	− 0·0024
J − K − L = n_9	+ 0·0024
K − L − M − N = n_{10}	+ 0·0008
L − M − N = n_{11}	− 0·0110
M − N = n_{12}	+ 0·0002
N = n_{13}	− 0·0132

It will be observed that the largest residual is only 0·0158 mean bridge-wire unit, or about 0·00003 ohm, showing that when all the contacts are kept clean the uncertainties due to variability of the contact-resistance are exceedingly small, and that the form of contact-maker employed is extremely constant in its action.

The largest residual observed in any standardization was 0·00008 ohm, and the average was about a quarter of this amount.

XIX. *Changes in the Resistance-Coils.*

In order to give an idea of the magnitude of the changes which took place during the work, we give in the following table the values obtained for the wire coils in the first and last standardizations expressed in mean bridge-wire units. In the fifth column is shown the change which took place in each coil, not taking into account the variation of unit. The figures were obtained by dividing the values in the fourth column for the several coils by the corresponding values in the third. The absolute magnitude of the changes cannot be deduced with certainty, but from other experiments made by one of us with manganine wires it seemed probable that the total change in any coil is in reality a combination of two distinct effects, the one being a change in the specific resistance of the wire throughout its entire length, and the other an effect confined to a small length at each end, which was very strongly heated during the operation of hard-soldering it to the copper tags.

Examination of the appended results shows that the change in the value of the lower coils is relatively much greater than in the case of the higher ones. This is in accordance with what we should expect, if the statement above were true, and both changes tended in the same direction.

	Nominal value in ohms.	Value No. 1.	Value No. 4.	$\frac{4}{1}$.	
A	40	18761·17	18763·71	1·00014	Wound on glass tubes.
B	20	9391·12	9393·25	1·00023	
C	10	4697·67	4698·80	1·00024	
D	4	1872·63	1874·01	1·00074	
E	3	1406·88	1407·39	1·00036	
F	2	939·276	939·562	1·00030	
G	1	462·770	463·047	1·00060	
H	·4	192·719	192·880	1·00083	Hanging free in air.
J	·3	146·367	146·610	1·00166	
K	·2	99·444	99·616	1·00172	
L	·1	48·498	48·627	1·00265	

XX. *Determination of the Temperature-Coefficient of the Coils.*

Preliminary determinations had shown that the temperature-coefficient of the wire used for the coils was extremely small, and had we been able to keep the box-temperature anywhere near constant we would hardly have needed to take it into account at all. As, however, considerable variations of the temperature of the room were inevitable, as previously explained, a method had to be devised to determine the coefficient with considerable accuracy. It had been previously found by one of us that the annealing process, to which the wire must be subjected to minimise subsequent time-changes in its resistance, has an appreciable effect on the temperature coefficient of the wire. In nearly all the specimens examined, the point where the characteristic change in sign of the temperature-coefficient takes place was displaced so as to occur at a lower temperature.

In view of uncertainties in the method of subjecting a sample of the wire to treatment *exactly* similar to what the coils themselves had received, and determining the coefficient of this piece—the process usually followed—we attempted to measure directly the actual coefficient of the coils themselves *in sitû*.

To do this we first tried a method consisting in the determination of the apparent value in box-units of a resistance kept at constant temperature, while the box-temperature was varied between that of the tap-water circulated through the outer tank and a maximum of about 35°. During these determinations every care was taken that the temperature of the coils as registered really represented their mean temperature at the time. Without going into details as to how this was attained by keeping up a continuous circulation of hot or cold water in the outer tank, and other precautions, we may say that the results of the measurements made were somewhat unsatisfactory, and the reason of this was traced to a curious and, we believe, not previously observed behaviour of the alloy in *not taking up instantaneously the resistance corresponding to a new temperature to which i may be subjected, especially when* *cooling.* We found that if the results of the separate determinations of the value of a constant outside resistance made with a series of steady box-temperatures with temperature *rising* be plotted, along with those of a series similar in every respect but with the temperature *falling*, the two do not overlap but form a loop. After a determination commenced at about 15°, during which the resistance-box was heated to 31° and allowed to cool, the whole temperature change occupying about nine hours, the coils did not return to their original resistance at 15° till they had been at this temperature about three days. We satisfied ourselves that this was due to a real lag in *resistance* and not in the indications of the box thermometers. The whole hysteresis effect is, however, small, and is quite imperceptible if the temperature changes are very slow, like the variations of laboratory temperature to which the box was ordinarily subjected. We may say that the temperature coefficient of the sample with which we observed the effect is rather abnormally small even for manganine, and that we had not time to see if the same effect could be observed with other specimens.

Although from the values thus obtained we might have deduced the temperature coefficient, using only the determinations made after a rise of temperature, we considered it advisable to make some fresh experiments, using a modification of the same method. During the first series of observations with thermometer K.8, a considerable number of zeros had been taken during a period when the box-temperature differed markedly from day to day. The thermometer had meanwhile never been disconnected from the box; the contacts remained in the same condition throughout, and we have no reason to believe that any secular change occurred in the leads or thermometer wire during the experiments. These experiments, during which the box temperatures ranged from 6°·60 to 19°·65, were accordingly utilised to calculate the temperature-coefficients of the coils, and from them a formula was obtained by least squares for the change of resistance of the box-coils with temperature.

Choosing as a standard temperature 15°, a table was calculated giving the coefficients by which the nominal box-resistances must be multiplied to give the true resistances. This multiplier is alluded to subsequently as the "factor" in the example of the method of calculating an experiment given later. The following numbers extracted from the table, show the magnitude of the coefficient in question :—

Temp.	Factor.	Temp.	Factor.
5°	1 − ·0000602	13°	1 − ·0000169
6	− 569	14	− 087
7	− 530	15	1 + ·0000000
8	− 485	16	+ 093
9	− 434	17	+ 193
10	− 377	18	+ 299
11	− 314	19	+ 410
12	− 244	20	+ 528

The coils used for this experiment were those lettered C and E, 10 and 3 ohms respectively, which may be taken as fairly representative. They enter into nearly all the comparisons with K.8.

It may be pointed out here that the influence of the variations of box-temperature on the results is largely eliminated in the experiments, as the fundamental points of the platinum thermometers were determined before, during, and after each series of experiments, and if a wrong value were taken for the coefficient to reduce all these to standard temperature, the errors committed would practically compensate one another.

We may mention here that the coefficients deduced by the method described above show a satisfactory agreement with those found from the *ascending* series of observations made by the first method, although the coils used in the two cases were not exactly the same.

XXI. Fixed Points of the Platinum Thermometers.

Before the commencement of each series of platinum thermometer comparisons a set of determinations of the zero and steam-points, generally about six in number, was always made; frequently check determinations of these points, especially of the zero, were interspersed between the comparisons themselves, thus giving an indication of the exact time when changes, if any, really took place. The zeros were taken in an apparatus similar in all respects to that described later in treating of the gas thermometer.

A few of the first steam-points were taken in an early form of the boiling-point apparatus usually employed at the Breteuil Laboratory, originally designed for mercury thermometers. During a long series of preliminary control comparisons between the platinum and gas thermometer at 100° we found, however, a very small but systematic discrepancy in the results, which disappeared when the steam-points of both thermometers were taken in the same apparatus. We therefore arranged that the same steam-point apparatus should be used by both of us in all the subsequent experiments.

The apparatus for the determination of the boiling-point of sulphur, and the special experiments made with it, are described later on p. 97.

XXII. Heating of the Thermometer Wire by the Current.

It is manifest that however small the current employed in the thermometers may be, it must needs heat them to some extent. Although the amount of this heating would be difficult to calculate, yet we thought it advisable to make a few experiments with a view to determine it, and at the same time to get some data from which we might be able to fix upon the best magnitude of the current to be employed for the thermometric measurements.

For this purpose we made a number of determinations of the apparent resistance of thermometer K.8 in ice using different battery-currents.

For these a curve was constructed showing the increase in apparent resistance of the wire with increasing energy absorbed in the coil, and a value calculated for what the resistance would be, if the current through it (and, consequently, the heating effect) were vanishingly small. Our measurements conclusively showed that, within the limits of accurate experiment, the heating effect was directly proportional to the watts in the wire, and that the heating per milliwatt for K.8 was about 0°·006.

In some of the earlier experiments, made before the heating effect was investigated, we employed a total resistance in the battery circuit of 150 ohms for measurements at 0°; the heating due to the current in this case being 0°·024. For all the subsequent experiments, however, by increasing the external resistance the heating was diminished to 0°·014 in ice.

Although we only made direct determinations of the magnitude of the heating effect at 0°, we have assumed, in the absence of further data, that for a thermometer coil the heating due to a given amount of energy expended in it is the same at all temperatures. As this is only approximate, some of our results may subsequently require small modifications; but the value we give later for the boiling-point of sulphur would not be affected, as it is expressed on the scale of the gas thermometer, the platinum thermometers being only used as an intermediary.

We calculated a table for each of the principal platinum thermometers, giving the resistance to be inserted in the external circuit for different temperatures.* In the example of a platinum temperature calculation given later, this number is referred to as the battery resistance "B.R. = 317 ohms."

XXIII. Determination of the Centre of the Bridge.

The index-error of the scale was determined from time to time during the work by reducing the resistances between C_1C_2 and P_1P_2 (fig. 1, p. 41) to zero, fixing all the contact pieces firmly in position, and determining the point of balance of the bridge. Should this not fall strictly at the centre of the scale, a correction for "bridge-centre" is applied in each resistance measurement.

* It was afterwards found that the formula used to calculate the table referred to was not strictly correct, but made the external resistance at high temperatures greater than it should have been. As, however, the total current heating at 0° was only 0°·014, and less than this at higher temperatures, the correction to be applied to the results, on account of the adoption of wrong external resistances, is probably well within the limits of experimental error, especially seeing that the error introduced is already partly compensated by its effect on the fundamental intervals as well as on the platinum temperatures found.

[Paragraph added December 1, 1899.—The measurements by which we attempted to determine the scale of the platinum thermometers may be divided into four groups, in which different instruments and means of heating were employed, and in which the precision varied from group to group.

These are—

(1) Comparisons in water between 0° and 50° of platinum thermometers K.8 and K.9 with the four principal mercury standards of the Bureau.

(2) Comparisons of K.8 and K.9 in an oil bath at temperatures between 80° and 200°, with a constant volume nitrogen thermometer, the initial pressure of the gas being 793 millims. of mercury.

(3) Comparisons of thermometers K.8 and K.9 between 250° and 460° in a bath formed of a mixture of nitrates of potash and soda, with the nitrogen thermometer, the initial pressure being 529 millims.

(4) Comparison of thermometer K.2 with the same nitrogen thermometer in the same bath between the temperatures 424° and 586°, the initial pressure being 392 millims.

As the sensibility of the gas thermometer varies according to the initial pressure, it is evident that the same precision cannot be attained in the different series. The construction of our instrument was such that the highest measurable pressure was about 1400 millims.]

XXIV. General Considerations on the Gas Thermometer.

In accordance with the decision of the International Committee of Weights and Measures,* the provisionally accepted normal scale of temperature is that of the constant-volume hydrogen thermometer. The employment of hydrogen for our work seemed therefore advisable, and before proceeding to the actual comparisons, we made a number of trials of the hydrogen thermometer between 100° and 200°. Up to temperatures about 180° these experiments gave fairly good results, but we noticed that prolonged heating above 180° was generally followed by a diminution of the gas contained in the thermometer reservoir. This diminution, though small, being regularly reproduced after each prolonged heating, might become serious at higher temperatures. Some special measurements, made on a known quantity of hydrogen enclosed in a capillary of "verre dur" of 1 square millim. cross section, and exposed

* The resolution fixing this was passed by the International Committee on October 15, 1887, and is as follows:—

"That the International Committee of Weights and Measures adopt as the Normal Thermometric Scale for the International Service of Weights and Measures, the centigrade scale of the Hydrogen Thermometer having as fixed points the temperature of melting ice (0°), and that of the vapour of distilled water in ebullition (100°) under the normal atmospheric pressure; the hydrogen being taken under the manometric initial pressure of one metre of mercury, *i.e.*, at $\frac{1000}{760} = 1{\cdot}3158$ of the atmospheric pressure."

repeatedly to temperatures varying from 200° to 250°, showed that the volume of the gas regularly diminished.

It therefore seems evident from these experiments that the walls of "verre dur" absorb a minute quantity of hydrogen.

It appears probable that this absorption is due to the reduction of sulphates contained in the glass. The employment of lead-glass as the material for the reservoir instead of "verre dur" would probably give rise to still more serious effects on account of the reduction of the salts of lead.

To avoid in the measurement of temperature the uncertainties caused by the variations of the gaseous mass, of which we have just spoken, and which might affect not only its quantity but its composition, we have substituted *nitrogen* for hydrogen. The nitrogen scale certainly diverges a few thousandths of a degree from the hydrogen scale in the interval 0° to 100°. Its departure from the normal scale at high temperatures is likely to be small and can always be corrected subsequently, when the necessary data have been collected.

The initial pressures of the *nitrogen* gas thermometer show no diminution, but rather a slight increase, which is explained by the contraction of the glass due to the annealing.

XXV. Comparisons of the Platinum Thermometers K.8 and K.9 with the Mercury Standards.

The direct comparison of the platinum thermometers with the large normal hydrogen thermometer between 0° and 100° would have necessitated such an enormous amount of work, without offering any special advantage, that we decided not to employ this instrument, but to take instead the four primary mercury standards of the Bureau, Tonnelot thermometers Nos. 4428, 4429, 4430, and 4431, whose corrections to the hydrogen-scale have been previously determined with all possible precautions by one of us. An account of this work is given in vol. 6, 'Trav. et Mém. du Bureau International.'

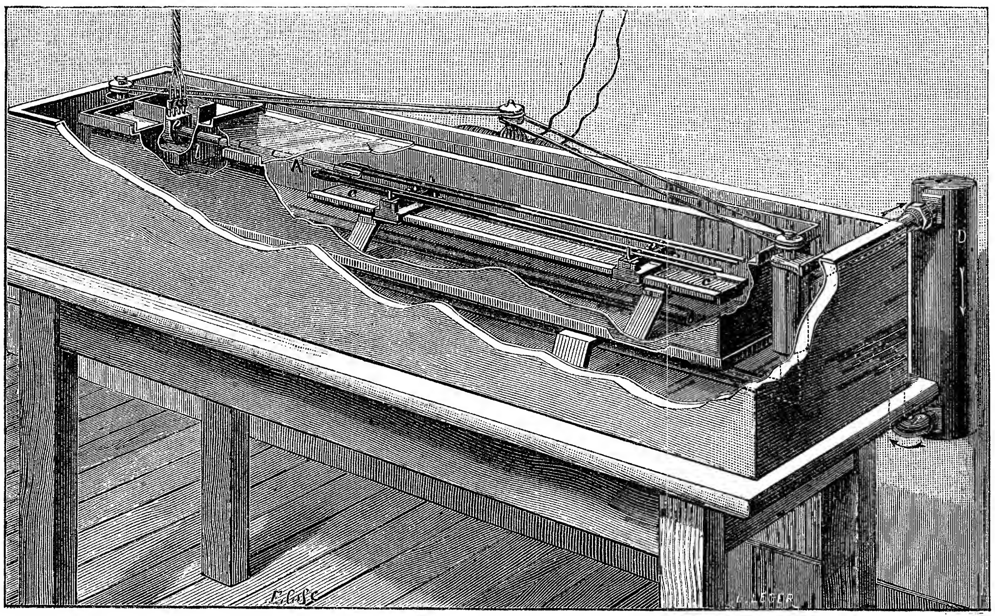
The comparisons between these mercury standards and the platinum thermometers were made in an apparatus constructed originally for the comparison of mercury thermometers with each other, which was modified and considerably improved for the purpose of this research. This apparatus is shown in fig. 11.

It consists of two concentric, rectangular, copper troughs; the outer one, which is protected by an oak case, having a capacity of about 70 litres. This trough communicates by a side tube with a small vertical copper vessel, well protected against radiation, which can be heated by a large gas burner. A screw stirrer, worked by a small motor, drives through the heater a rapid current of water, which is taken in at the opposite end of the trough by a horizontal tube resting on the bottom, and circulates as shown by the arrows in the figure.

The interior trough, which is 112 centims. long, 17 centims. wide, and 14 centims. deep, is provided at one end with a system of screw blades for stirring. Resting upon its bottom is the metal framework on which the thermometers are arranged. This thermometer support is so contrived that all the thermometers can, without risk of straining them, be simultaneously clamped parallel to one another, and in the same horizontal plane.

During the comparisons the platinum thermometer was fixed horizontally, with its spiral in the same plane as the mercury thermometers, and close to them. To prevent the water from penetrating to the portions of it which were exposed, the head

Fig. 11.



Horizontal Bath for Comparisons in Water.

A, stem, and *e*, head, of platinum thermometer; *l*, brass box surrounding the head of platinum thermometer; *D*, heater for water in outer tank; *c*, plate of milk glass.

was placed in a square brass box open above and provided with a side tube, through which the greater part of the length of the thermometer stem projected, the joint being made by an india-rubber stopper.

The internal tank is provided with a rim, on which rests a piece of plate glass 8 millims. thick, covering the whole surface of the water, with which it is just in contact.

By this arrangement the cooling by evaporation is almost entirely prevented, and the attainment of a very constant temperature much facilitated. When operating at temperatures below that of the room, it is advisable to cover the glass with a thin layer of water, in order to avoid the deposition of dew upon it.

The readings of the mercury thermometers are made by means of a small vertical reading telescope sliding on the glass plate, and can be made while the stirrer is at work. The space around the inner tank is closed above by a metal lid, pierced with the necessary openings for the axes of the stirrers.

Observations with Thermometer K.8.

The thermometer K.8 was compared under these conditions with the four standards at temperatures between 0° and 50°. These experiments are numbered 4 to 17 in the summary of results for K.8 at the end. To avoid the errors of parallax on the mercury thermometers, ten readings were made with the divisions upwards and ten with the thermometers turned through 180°. After each series the zeros of the mercury thermometers were observed in the usual manner with a micrometer telescope.

Three other observations at higher temperatures (numbered 18 to 20 in the table) were made with the same thermometers in another apparatus, described later when treating of the gas thermometer. In the three experiments all the instruments were used in the vertical position.

Care was taken to have only a very small emergent column in each case. The bath was filled with oil, and was heated by the vapour of ethyl alcohol boiling under various pressures.

We also made three measurements below 0° in an apparatus specially constructed for experiments at low temperatures, which has been described in the 'Procès-verbaux des Séances du Comité International,' 1891, page 33. The thermometers plunge into a bath of alcohol cooled by the evaporation of liquid methyl chloride, and stirred continuously by a suitable mechanism. The two mercury standards, Tonnelot thermometers Nos. 11,165 and 11,166, which were employed for these observations, have been studied at the Bureau, and compared directly with the hydrogen thermometer under the same conditions.

The series we made consists of three experiments numbered 1 to 3 in the Summary.

Observations with Thermometer K.9.

The later series of comparisons of thermometer K.9 with the mercury standards was made under precisely similar conditions to those described above for K.8, but the number of different points in this case was not so great; each experiment consisted of only ten observations instead of twenty as before. The experiments between 0° and 50° are numbered 1 to 6 in the table.

XXVI. Description of the Gas Thermometer.

The gas thermometer we employed for our researches is similar to the instrument previously described by one of us* and is shown in fig. 12.

It is a constant-volume thermometer arranged so as to permit the determination of the total pressure of the gas contained in the thermometric reservoir *by a single reading*. The simplification thus introduced into the measurements permits rapid observations without sensibly diminishing the precision, which is limited more by the conditions of uniformity of the baths and other heating arrangements employed than by reading errors.

The apparatus constituting the gas thermometer is installed on a foundation of concrete about 1 cubic metre in volume abutting against one of the massive walls of the laboratory. On this foundation on the left hand rests a rectangular stone pillar, with slate top, carrying the manometric apparatus, while the heating baths on the right are supported on the same foundation at floor level, this however not being shown in fig. 12. The manometric apparatus is protected from heating by a large paper screen which extends upward right to the ceiling, and which is pierced with the holes necessary to admit the passage of the various connecting portions.

(a) *Thermometric Reservoir.*

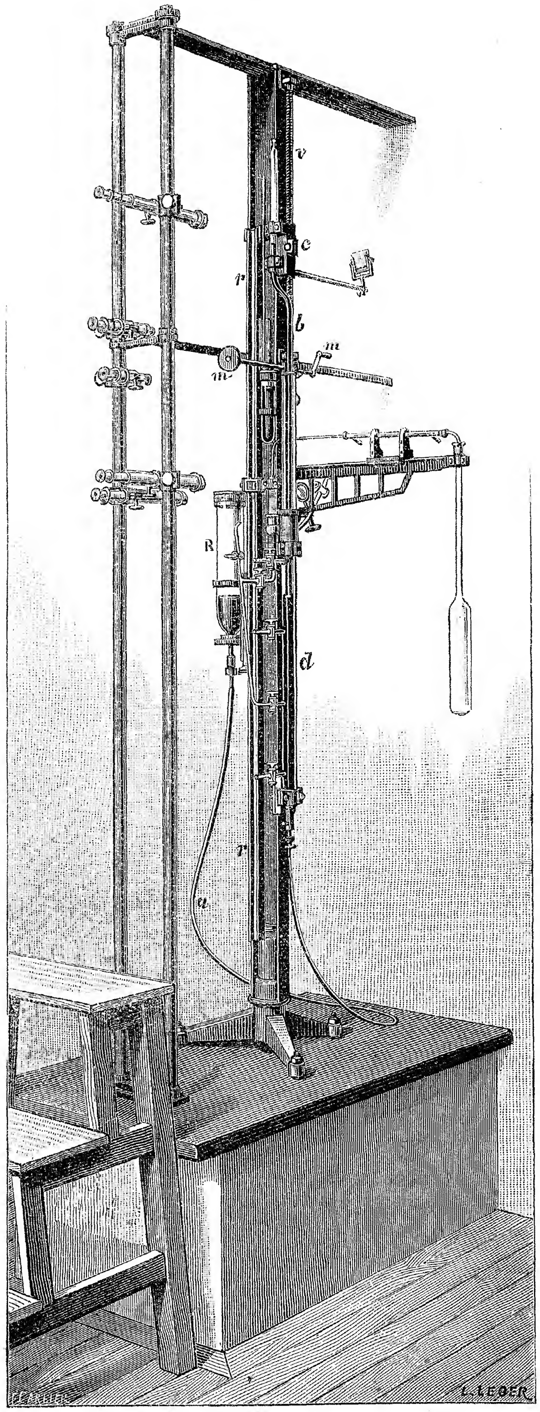
For the first part of the experiments we employed a cylindrical reservoir of hard glass drawn from a tube 36 millims. external diameter and 1·5 millims. thick.

This tube, closed at one extremity, was fused at the other to a capillary tube of the same glass, having a bore of 0·53 millim. and 90 centims. long. Fig. 12 shows the thermometric reservoir mounted for the experiments, its axis being in the vertical position, which we find the most convenient for the introduction of the reservoir into the various baths employed. The outer bend of the tube carrying the reservoir is supported by a light frame from the iron girder carrying the manometer, and slides vertically along this through a considerable distance.

The porcelain reservoir employed for work at high temperatures was obtained from the Royal Porcelain Factory at Berlin. It is cylindrical in shape and is 36 millims. external diameter and 20 centims. long. The reservoir has a neck 28 centims. in length and 11 millims. exterior diameter. It is pierced with a hole of 2 millims. diameter, into which just passes the platinum capillary uniting the reservoir to the manometer. The joining of the platinum tube to the neck of the manometer is a matter of some difficulty, since it is of great importance that this joint should be absolutely gas-tight. We finally adopted the following disposition which has answered well. On the platinum tube which enters to a length of 11 centims. into the neck of the reservoir, a brass washer is soldered which fits exactly to the flat end of the

* 'Trav. et Mém. du Bureau International,' vol. 6, p. 28.

Fig. 12

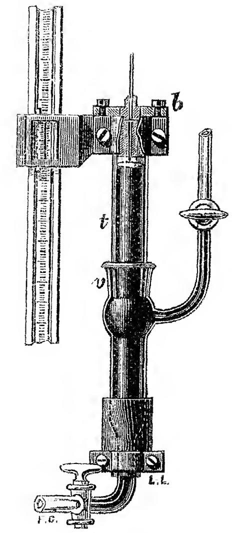


Sketch of Gas Thermometer.

The reservoir is shown raised above its natural position in the comparison bath, and the screens are all removed.

R, mercury reservoir; *a*, steel connecting tube; *r*, *r*, scale; *c* and *v*, slide and worm gear holding the barometer and scale; *m*, *m*, handles for adjustment of position of barometer and scale.

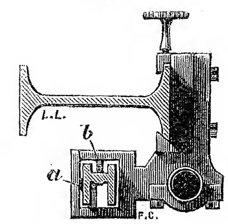
Fig. 13.



Detail of Manometer-point.

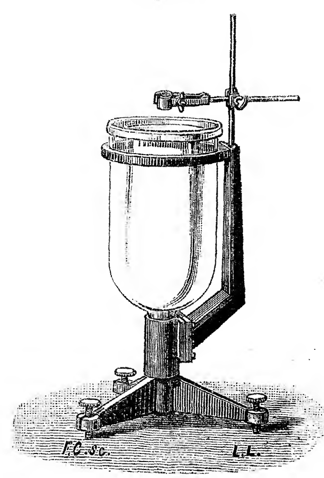
t, glass tube optically ground inside and out; *b*, brass collar, holding the cylindrical stopper of nickel-steel; *v*, ground joint.

Fig. 14.



Section of slide carrying the manometer and scale.

Fig. 15.



Zero apparatus.

neck. This washer is tightly held in place by a brass clamp, which screws on to a collar made in halves and fixed to the narrow part of the neck by a cement, which is a little less fusible than that employed for the joint proper.

(b) *Barometer and Manometer.*

The manometric apparatus is mounted on a vertical iron girder, 2 metres high and of H-shaped cross-section, solidly bolted by three diverging iron feet to the massive pillar previously described. The external faces of the girder are planed up as true as possible over their entire length, and on them slide supports for the manometric tubes, the barometer, and mercury reservoir. To increase the stability of the whole, the top of the iron column is fixed to the wall by a transverse piece, which also serves to support two brass tubes on which slide the observing telescopes.

The barometer consists in its upper part of a tube of 15 millims. internal diameter. A point of black glass is fixed axially in the interior of this tube by fusion. This is referred to subsequently as the *barometer-point.* Below the barometric chamber the tube has a double bend, which brings the lower part of the tube 4 centims. to the right of the upper portion.

The barometer tube is firmly fixed above to a carriage, *c*, which can be displaced vertically by the movement of a screw 60 centims. in length, working in bearings above and below, and engaging by bevel gearing with a horizontal shaft projecting forwards. By turning the small handle *m* the barometer can be raised or lowered at will.

The piece which maintains the barometer tube on its support also carries suspended between two points by one of its extremities a graduated brass scale 1·5 metres long, whose axis is at a distance of 48 millims. from the barometer point. This scale shares all the movements of the barometer carriage, and the glass point may be assumed to have an invariable position with reference to the neighbouring divisions of the scale.

The lower end of the barometer is immersed under mercury in a tube of 90 centims. in length and 25 millims. in diameter, which serves as its reservoir ; this tube can be fixed at different levels on the manometer support.

Projecting from the front of this tube are four stop-cocks at intervals of 15 centims., serving to establish communication between the barometer and manometer at any height.

The open branch of the barometer communicates below by means of a long steel tube, *a*, with a reservoir, R, of large capacity filled with mercury, which can be displaced vertically either rapidly by hand or slowly by a micrometer screw.

(c) *Manometer.*

The closed branch of the manometer, the details of which are shown in the fig. 13, is composed of a rather thick-walled flint glass tube 16 millims. internal diameter, which has been optically worked inside and out, in order to render it perfectly cylindrical; the freedom from longitudinal striæ thus obtained gives great sharpness to the images obtained through it. The upper end is closed by a stopper of glass or metal, pierced with a fine hole. Into this stopper is fastened the end of the capillary tube fused to the thermometric reservoir. The stopper, which is ground perfectly cylindrical, enters the tube, which it fits closely, for a length of 25 millims., and is fixed in position by a very thin layer of Canada balsam, thus forming a perfect joint.

The lower part of the stopper is plane and well polished, and carries at its centre a very fine steel point 0·5 millim. long, which serves as an index mark to which the mercury may be accurately adjusted. To avoid all displacement of the stopper and tube in their support, a brass collar is fixed in a groove ground in the stopper, and this is firmly screwed to the iron support by the clamp *b*. The piece of bronze carrying the manometer tube also serves to maintain the position of the lower end of the scale, and to carry the vernier, whose zero thus occupies an invariable position with regard to the steel point in the manometer tube. The closed limb of the manometer is so arranged on its support that its axis is in the same vertical line as the point in the upper chamber of the barometer. The scale remains vertical for all positions of the sliding supports of the barometer and manometer. These conditions being fulfilled, it is evident that if the distance between the point in the closed branch and the zero of the vernier is once for all known, a single reading of the scale, corrected for the "index error," which is defined later, suffices to give the difference of level between the two points.

The closed branch of the manometer fits below into a glass T-piece, the horizontal limb of which communicates with a system of tubes serving for the exhaustion and filling of the reservoir. The lower end of this glass tube is bent horizontally forwards, and communicates by a tap with one of the four taps on the open branch of the manometer.

(d) *Measurements of Pressures.*

The disposition of the manometric apparatus permits, as has just been seen, the measurement at any moment of the distance between the two points in the barometer and the closed branch. The communication between the columns of mercury filling the manometer and the barometer reservoir being established, the pressure exerted by the gas on the mercury in the closed branch is balanced by increasing or diminishing the height of the mercury in the open branch, which is effected by raising or lowering the auxiliary reservoir placed on the left. The barometer tube is simultaneously displaced, in order to keep the mercury in the neighbourhood of the point in the barometric chamber. The equilibrium sought is attained when the mercury just reaches at the same time the points in the closed branch and in the barometer chamber. The observation of this adjustment of the mercury is made by means of two small telescopes magnifying about 36 times, sliding vertically on a brass tube and placed at a distance of 38 centims. from the manometer. A second tube serves to support three other small telescopes, for the observation of the scale-vernier, and two auxiliary mercury thermometers, which are placed close to the mercurial columns to indicate their temperature.

During the measurements the observer is at a distance of about 50 centims. from the apparatus; his influence on the temperature of the mercurial columns is thus considerable, as is also that of the various heating baths, and by reason of the great expansion of mercury, this heating probably constitutes one of the principal sources of error in the experiments. To diminish as far as possible the radiation from the comparison-bath, a double walled metallic screen, in which a current of water circulated, was interposed between it and the manometer.

(e) *Divided Scale.*

The divided scale used was constructed by the Société Génévoise, and has served for previous work with the gas thermometer. Its length is 1·5 metres, and its cross-section is in the form of an H. This H-form is not well adapted for use with a vernier. It would be better to adopt a form allowing the surfaces carrying the divisions of the scale and vernier to be in the same plane. In this scale the divisions are on a plate of silver let into the median transverse face, very near the plane of the neutral fibres.

Fig. 14 shows in horizontal projection the disposition of the pieces which support the scale and attach it to the barometer, and fig. 13 the pieces which hold the vernier on the support of the manometer tube, and which ensure contact between the scale and the vernier. The method of suspension of the scale permits it to turn about two axes perpendicular to its length. In the two directions of free movement two springs gently press the scale against the vernier.

Two thermometers placed at equal distances from the points in the barometer and manometer tubes, the one on a fixed support, the other on an attachment to the barometer, serve to indicate the temperatures of the mercurial columns and of the scale. Each of these thermometers is placed in a test-tube filled with mercury, of the same diameter as the neighbouring portion of the manometer tubes. This symmetrical arrangement of the thermometers with regard to the ends of the mercury columns whose temperature is to be measured considerably simplifies the calculation of the mean temperature of the manometer.

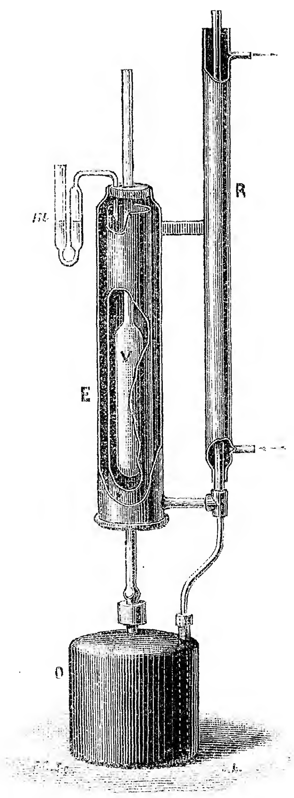
XXVII. Zero Apparatus.

A glass bell-jar supported on an iron tripod and surrounded by several layers of felt serves as the receiver. The ice, finely divided and saturated with pure water, is pressed around the reservoir of the thermometer, the emergent stem being held by a clamp fixed on the support of the bell-jar. The apparatus, filled with ice and protected by a cover of thick flannel, can be left three or four hours without the least perceptible change of temperature in the central part occupied by the reservoir.

XXVIII. Steam-point Apparatus.

The 100-point apparatus, shown in fig. 16, is composed of a small boiler of 3 litres capacity, communicating by a lead tube with a double walled vertical vessel, into

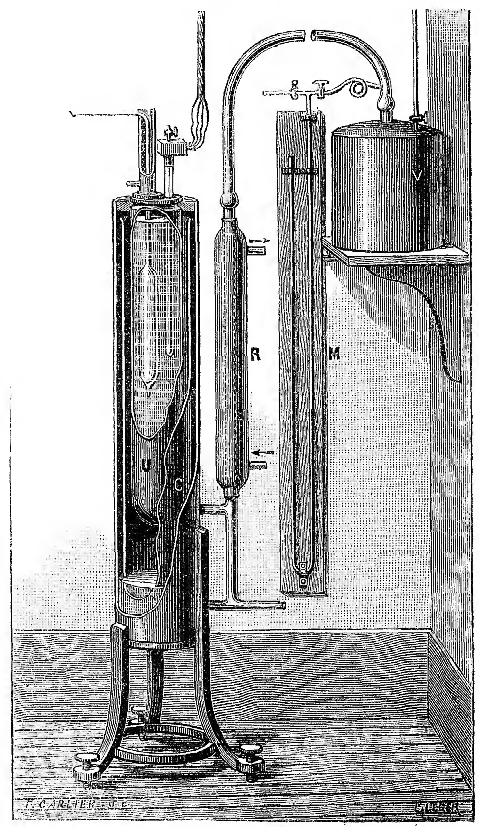
Fig. 16.



Steam-point apparatus.

V, reservoir of gas thermometer.
O, copper boiler.
E, double-walled cylinder.
R, condenser.
m, water manometer.

Fig. 17.



Oil-bath for comparisons to 200°.

Note.—The stirring arrangements are not shown.
U, copper oil vessel.
R, condenser.
V, air reservoir.
M, manometer.
C, wall of vapour bath cut away to show interior.

which the thermometer reservoir can be introduced from above. The vapour developed in the boiler first passes up the inner tube of the steam bath, then descends by the exterior annular space, finally arriving at the condenser, whence it returns to the boiler by a tube plunging below the water-level. All the communication tubes between the different parts are wide, and arranged so as to avoid the possibility of their becoming choked by the condensation of water in them. The excess of the interior pressure over that outside can be measured by a small water-manometer introduced into the cork.

XXIX. Comparison-bath for Range 80° to 200°.

The disposition of this apparatus, indicated in fig. 17, is the result of a long series of experiments, the aim of which was to obtain a bath sufficiently uniform in temperature to be employed for the accurate comparison of mercury thermometers with the gas thermometer over the range mentioned. It fulfils satisfactorily the principal requirements of an apparatus of this kind, viz. :—

(1) Uniformity of temperature throughout a space of large dimensions.

(2) Rapid re-establishment of a steady state after the pressure in the boiler has been altered.

(3) Employment of a small number of inexpensive liquids easily obtained in a state of sufficient purity.

The boiler consists of a cylindrical vessel of planished copper 2 millims. in thickness; it has a diameter of 17 centims. and a height of 82 centims. A bell-shaped vessel of the same material is soldered by its rim concentrically into the interior of the cylinder. This inner vessel is filled with a heavy petroleum oil, in which the reservoirs of the thermometers to be compared are directly immersed.

The stirring is effected by a vertical stirrer (not shown in the figure), the stems of which emerging from the bath are protected against cooling by glass tubes. The annular space between the two vessels serves for the circulation of the vapour, and to increase the uniformity of this circulation the space is divided into two approximately equal parts by the introduction of a thin tube of copper open at both ends, and resting on the bottom of the cylinder. The vapour given off by the boiling liquid, which fills the lower part of the outer vessel, rises first in the interior space in contact with the walls of the oil-bath, then descends by the exterior, again ascending into the condenser placed at one side, whence it passes in the state of liquid back to the boiler by a lateral tube. The reversed condenser is in communication by a wide tube with a large copper reservoir in which the pressure can be varied at will, or kept constant, thus changing the temperature of ebullition by a considerable amount; by using only three liquids any temperature between 80° and 200° can be quickly attained and kept extremely constant for any length of time, provided only that the joints in the whole apparatus remain perfectly tight. A mercury manometer indicates the pressure of the vapour. The bath is covered with several layers of asbestos-card to avoid losses by radiation and their effects on the temperature of the room.

XXX. Preliminary Determinations.

(a) *Measurement of the Capacity of the Thermometric Reservoir.*

Before proceeding to the measurement of the capacity of the thermometric reservoir, we considered it advisable to subject it to a prolonged annealing at the temperature of the boiling-point of sulphur. After thirteen hours' heating we obtained for its capacity at 0°

$$V_0 = 159 \cdot 670 \text{ cub. centims.},$$

and after a second exposure of eleven hours we found

$$V_0 = 159 \cdot 642 \text{ cub. centims.}$$

This value was a little modified during the operation of mounting the thermometer, as a short piece of the connecting capillary had to be suppressed. Allowing for this we found for the first part of the experiments the value

$$V_0 = 159 \cdot 629 \text{ cub. centims.}$$

(b) *Coefficient of Dilatation of Hard Glass.*

The dilatation of "verre dur" was not measured directly on the thermometric reservoir itself, but on a tube of 1 metre length drawn from the same melting; its linear dilatation was determined by a long series of experiments at temperatures comprised between 0° and 100°. These experiments have given for the law of cubic dilatation of glass between 0° and 100° the formula

$$V_t = V_0 (1 + 0 \cdot 000\ 021\ 801\ t + 0 \cdot 000\ 000\ 015\ 536\ t^2),$$

whence, for $t = 100°$,

$$V_{100} = V_0 (1 + 0 \cdot 002\ 335\ 50).$$

During some subsequent experiments on the effect of prolonged heating on glass, we had occasion to control this result by determining the dilatation between 0° and 100° of a "verre dur" vessel drawn from the same tube as the thermometric reservoir. From these observations we found

(1) Before annealing . . . . $V_{100} = V_0 (1 + 0 \cdot 002\ 357\ 1)$,

(2) After annealing at 445° for 81 hours . . $V_{100} = V_0 (1 + 0 \cdot 002\ 343\ 6)$.

These last measurements furnish no indication of the magnitude of the term in t^2, which alone has any influence on the temperature measurements. We have therefore employed, for all the observations relative to the glass reservoir, the expression with two terms indicated above.

(c) *Determination of the Coefficient of Pressure of the Thermometric Reservoir.*

The capillary tube fused to the thermometric reservoir had, at the commencement, a length of about 250 millims. The volume of this tube was determined by weighing a thread of mercury occupying in it a length of 200 millims.; the weight of mercury contained in 1 millim. of length was found to be 1·69185 gramme. The calibre of the tube was afterwards studied by GAY-LUSSAC'S method between the two extreme points 0 and 250. The calibration corrections thus obtained had to be applied in the reduction of the observations on the coefficient of pressure. To measure this coefficient the same method is followed as for the determination of the coefficient of external pressure of mercury thermometers.

The thermometric reservoir is placed in a glass tube filled with water, and closed by a cork pierced with a hole, through which passes the capillary attached to the reservoir. The space between the reservoir and the external tube can, by means of stop-cocks, be put into communication either with the atmosphere, or with a large exhausted vessel. The reservoir itself being filled with water up to a certain scale division, observations are made of the displacements of the meniscus produced by varying the external pressure by nearly an atmosphere.

The observations effected under these conditions gave, after all reductions, the following value for the variation Δv of the volume of the reservoir, which corresponds to a variation of external pressure equal to a millimetre of mercury,

$$\Delta v = 0{\cdot}006\ 228 \text{ microlitre.}$$

This value of the coefficient of pressure was employed for the calculation of a table giving the variations of volume of the reservoir corresponding to the changes of internal pressure observed in the course of the experiments.

(d) *Determination of the "Dead Space" ("Espace Nuisible").*

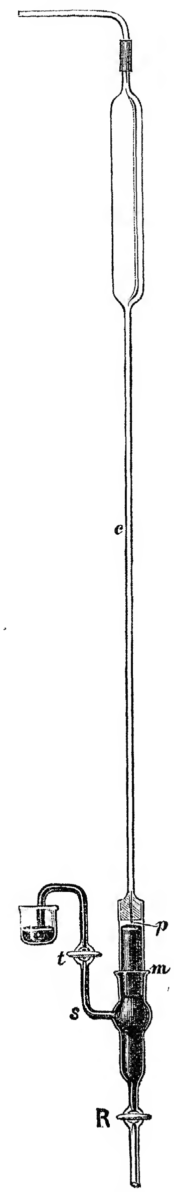
The determination of the volume of the space occupied by the gas not exposed to the same temperature as the reservoir presents peculiar difficulties. It is of extreme importance that the limits of this space should be well defined, which, however, cannot be done quite rigorously.

The "dead space" may be divided into two parts: (1) the space occupied by the gas in the closed branch of the manometer between the mercury touching the point and the lower surface of the stopper, and (2) the internal volume of the capillary tube between the plane of the stopper and the part of the tube which penetrates into the heating apparatus. The curvature of the mercury-meniscus in the closed branch is necessarily somewhat variable, and as the diameter of the tube is 16 millims., small variations in the capillary angle have an appreciable effect on the volume of the gas above the mercury. The extent of the second part on the side of the reservoir is also somewhat uncertain, because of the rapid variation of temperature near its end, but as the capillary tube has only a very small diameter, the influence of this cause of error is not great.

In order to avoid the uncertainty of any hypothesis concerning the capillary angle under the actual circumstances, we attempted to measure the total volume of the "dead space" directly in the following manner.

The capillary tube joining the thermometric reservoir to the manometer being straight, the closed branch of the manometer was fixed on its support in the position it afterwards had to occupy (fig. 18), and the open extremity of the reservoir was connected to the mercury pump. The side tube, S, of the manometer terminated in a tube bent downwards, whose lower end was about on a level with the point. The tap, R, was placed in communication with the auxiliary mercury reservoir forming part of the gas thermometer. The tube, *m*, was first exhausted, and then filled up with mercury to near the steel point, and the tube, *s*, completely filled; then air was readmitted, and the mercury was adjusted to the point by slightly displacing the reservoir. The taps, R and *t*, being then shut, the whole was again exhausted. A small, carefully weighed vessel containing mercury was placed under the tube, *s*, and by opening the tap, *t*, mercury was allowed to enter the manometric tube and fill all the space above the point, rising to the level, *c*, which is at the barometric height above the mercury in the weighed vessel. The volume of the "dead space" could then be deduced from the loss of weight of the small vessel. It should be remarked that the pressure in *m* at the end of the experiment was very nearly the same as at the beginning, and all uncertain corrections were thus avoided. The divergence found between the individual observations given are a fair measure of the inevitable variations of the "dead space" during the experiments.

Fig. 18.



Determination of the volume of the "dead space."

If the point, *c*, did not coincide exactly with the limits of the "dead space," it would be easy to take account of the difference, the volume of the capillary tube having been previously measured.

The following values for the volume of the "dead space" were obtained by the above method :—

	cub. centim.
(1)	0·4371
(2)	0·4503
(3)	0·4469
(4)	0·4380
(5)	0·4385
(6)	0·4385
(7)	0·4385
Mean . . .	$v =$ 0·4411

(e) *Determination of the "Index Error."*

The readings made on the scale by means of the vernier do not represent exactly the difference of level between the points in the barometer and the closed branch of the manometer; this is due to two causes. The first is that the point of the barometer is not at the same level as the division, 0, of the scale. All the scale-readings have, therefore, a correction applied which we will call the correction for "index error," which must be determined by special measurements with a good cathetometer. This correction, which would be constant if the plane surface of the girder on which the barometer slides were absolutely true, varies slightly according to the position of the barometer.

The second cause is that the point in the manometer is not at the same level as the division, 0, of the vernier. This latter correction may, perhaps, be considered constant for a given position of the closed branch of the manometer.

The correction for the "index error" has been determined frequently during the course of the experiments, especially that relating to the lower point, the position of which has been modified several times. The publication of the observations being of no interest, we give simply the values of the constants relating to two positions of the closed branch. From the observations of the 3rd and 4th of May, 1898, the correction relating to the barometer was found to be

$$C_b = -\ 9{\cdot}588 \text{ millims.},$$

and the correction relating to the manometer point in the raised position (for observations at 0° and 100°)

$$C_m = +\ 20{\cdot}130 \text{ millims.},$$

whence the total correction for "index error" is given by

$$C = C_b + C_m = 10{\cdot}542 \text{ millims.}$$

In the lower position of the manometer (used in comparisons between 100° and 200°) the total correction had the slightly different value $C = 10{\cdot}552$ millims.

(f) *Corrections of the Scale and Vernier.*

The corrections of the scale are given in vol. 6 of 'Trav. et Mém. du Bureau International.' We need say here only that the study of this scale by M. ISAACHSEN gives the corrections at each decimetre graduation, except the second, and at all the even centimetres between 500 and 1400.

The vernier is divided into twenty parts, and its total interval (0, 20) corresponds to a length of 18·980 millims. instead of 19 millims. A correction must therefore be applied to the vernier readings, which is proportional to the fraction measured, and whose maximum value is 20 micron.

The measurements made to verify the equidistance of the divisions of the vernier showed that the errors of division attain 10 micron. for certain lines, but by reason of their irregular distribution, and of the repetition of the observations using different parts of the vernier, they have not been taken into account.

XXXI. CALCULATION OF THE TEMPERATURES.

The deduction of the formula employed for the calculation of the temperatures has been given with all necessary details in the memoir already quoted,* therefore we only give here a résumé of the process.

Let V_0 be the volume at 0° of the gas contained in the thermometer reservoir;

δ the mean coefficient of dilatation of the reservoir between 0° and T°;

α the coefficient of expansion of the gas at constant volume;

v the volume of the "dead space" at the standard temperature $t°$;

Δv and Δt the variations of volume and temperature of the "dead space";

H_0 the initial pressure of the gas corresponding to the temperature 0° of the reservoir and $t°$ of the "dead space";

$H_0 + h$ the pressure of the gas at the temperature T° to be determined; the temperature of the "dead space" being $t + \Delta t$, and its volume $v + \Delta v$;

β_i the internal pressure coefficient of the thermometric reservoir.

The total mass of the gas being the same at the temperatures 0 and T, we have

$$\left(V_0 + \frac{v}{1 + \alpha t}\right) H_0 = \left[\frac{V_0 (1 + \delta T) + \beta_i h}{1 + \alpha T} + \frac{v + \Delta v}{1 + \alpha (t + \Delta t)}\right] (H_0 + h).$$

Suppose now that we have applied to the pressures H_0 and $H_0 + h$ the corrections necessary to reduce them to what they would have been had the whole "dead space" been maintained at 0°, and let us call these new pressures H_0' and $H_0' + h'$; we have then the simplified formula

$$(V_0 + v) H_0' = \left[\frac{V_0 (1 + \delta T) + \beta_i h}{1 + \alpha T} + v\right] (H_0' + h'),$$

* 'Trav. et Mém. du Bureau International,' vol. 6, p. 52.

whence, by certain simplifications, we get finally

$$\alpha T = \frac{H_0' + h'}{H_0'}\left[1 + \delta T + \frac{\beta_i h}{V_0}\right] + \frac{h'v}{H_0 V_0}(1 + \alpha T) - 1.$$

This formula was used first to calculate the coefficient α between the known temperatures 0° and 100°, the value found being afterwards utilised for the calculation of the temperatures observed in the comparisons.

XXXII. Corrections Relating to the "Dead Space."

The corrections, which must be applied to the observed pressures, to reduce them to what they would have been had the whole of the "dead space" been at 0° throughout, are easily deduced from the laws of Boyle and Gay-Lussac.

(1) Let us first suppose that the "dead space" is composed of different parts

$$v = v_1 + v_2 + v_3 + \ldots$$

whose temperatures are

$$t_1, t_2, t_3, \ldots .$$

If now we reduce to 0° these gaseous volumes without changing the pressure p to which they are subjected, we have as total variation of volume

$$\Delta v = \frac{v_1}{(1 + \alpha t_1)} + \frac{v_2}{(1 + \alpha t_2)} + \frac{v_3}{(1 + \alpha t_3)} + \ldots - v.$$

The temperatures t_1, t_2, t_3 . . . , being generally positive, Δv is negative.

(2) To find the correction sought, it is necessary to transfer from the reservoir, where the temperature is T°, and the pressure p, a quantity of gas occupying at 0° the volume Δv, or what comes to the same thing, the volume of the reservoir V must be increased by a quantity equal to $\Delta v\,(1 + \alpha T)$. It is evident that this increase in volume involves a variation of pressure

$$\Delta p = p\left(\frac{V}{V + \Delta v(1 + \alpha T)} - 1\right),$$

which can also be written

$$\Delta p = -\frac{\Delta v}{V}(1 + \alpha T)\,p,$$

and which represents the correction sought.

For the application of these corrections we constructed two tables. The first gives for every degree the values of

$$v_1\left(\frac{1}{1 + \alpha t_1}\right),\quad v_2\left(\frac{1}{1 + \alpha t_2}\right),\quad v_3\left(\frac{1}{1 + \alpha t_3}\right),\ \text{etc.},$$

and enables the values of Δv to be rapidly calculated.

The second table gives the values of

$$-\frac{\Delta v}{V}(1 + \alpha T)$$

for all the values of T in the comparisons.

XXXIII. Filling of the Gas Thermometer.

The nitrogen employed was prepared by the following method; into a solution of 100 grams of potassium bichromate in 900 grams of distilled water were introduced 100 grams of nitrite of soda and 100 grams of nitrate of ammonia. When gently heated, this mixture gives off a very regular stream of nitrogen, which is collected in a large bottle over distilled water. To destroy any oxides of nitrogen which the gas may contain, it was passed through two tubes containing caustic potash, then over copper, heated to dull redness in a combustion tube, and finally through a series of drying tubes containing baryta and phosphoric anhydride. The gas, after remaining a long time over the drying agents, was introduced into the reservoir of the gas thermometer by a series of glass tubes, leading on the one hand to the tap on the manometer-limb and on the other by a side-tube to the mercury pump.

The reservoir was then heated for some time to about 250°, being meanwhile thoroughly exhausted by the mercury pump. Dry nitrogen was then admitted, and the alternate evacuation and filling with gas were repeated several times. Our first definite filling was made on February 2, 1898.

As the comparisons were to extend between the limits 100° and 200° the initial pressure at 0° C. was adjusted to be approximately 800 millims. of mercury, the pressure at 200° corresponding to this being about 1,387 millims. This is nearly the highest pressure which can be measured on the manometer.

XXXIV. Determination of the Initial Pressure.

It is essential to measure repeatedly the pressure of the gas at the temperature of melting ice, in order to make sure that no leakage takes place at the joints, and to be in a position to take into account the inevitable small variations in capacity which take place when a glass reservoir is employed. As we have previously mentioned, prolonged heating produces a permanent contraction of the glass, therefore we may expect an increase in the initial pressure after the comparisons at high temperatures.

For observation of the initial pressure the zero apparatus, previously described, is used. We give, as an example of a determination, the second series of observations of May 3, with their reductions, in the form adopted throughout the whole of the experiments.

We have taken for the temperature of the "dead space" that indicated by the nearest thermometer, No. 4365.

3rd May.

Time.	Vernier readings.		Auxiliary thermometers of the manometer.	
			4365 (low).	4362 (high).
	millims.			
11.0	784·765		14°·960	15°·320
	·770		·950	·330
	·770		·930	·350
	·765		·940	·400
11.30	·770		·970	·430
Mean . . . . =	784·768		14·950	15·366
Vernier correction . . . =	+ ·015	Total correction . .	− ·054	− ·348
Correction for index error . =	+ 10·542			
	795·325		14·896	15·018
Correction for dilatation . =	− 1·934			
,, ,, gravity . . =	+ 0·263		Mean = 14°·957	
,, ,, the scale . =	− 0·026			
Δp . . . . =	− 0·114			
Initial pressure . . . . =	793·514		Δv = − 22·9 microlitres	

All the observations given on the following page have been calculated exactly as in the example quoted, regard being paid to the subsequent modifications in the "index error."

Date.	Initial pressure in millims.		Remarks.
February 3	793·413		
„ 4	·468		
„ 4	·404		
„ 5	·456		
„ 5	·421	793·421	From February 4 to 9 six determinations at 100°
„ 7	·427		
„ 8	·395		
„ 8	·395		
„ 9	·424		
„ 9	·406		
			From March 23 to April 2 comparisons with the thermometer K.8
April 22	793·516		
„ 22	·509		
„ 23	·525		
„ 23	·498		
„ 25	·528		
„ 25	·506		
„ 27	·511	793·519	From April 22 to 27 seven determinations at 100°
„ 27	·512		
„ 27	·522		
May 2	·543		
„ 2	·527		
„ 3	·532		
„ 3	·514		
			From May 6 to 10 seven determinations at 100°
May 11	793·557		
„ 11	·526		
„ 11	·543	793·539	
„ 12	·536		
„ 12	·534		
			From May 14 to 24 comparisons with the thermometer K.8
May 26	793·553		
„ 26	·544		
„ 27	·575		
„ 27	·557		
„ 27	·548	793·563	From May 26 to 28 five determinations at 100°
„ 28	·563		
„ 28	·580		
„ 29	·569		
„ 29	·582		
			From May 30 to June 2 comparisons with the thermometer K.9
June 3	793·589		
„ 3	·574	793·566	
„ 4	·546		
„ 4	·554		

XXXV. Determinations of the Coefficient of Expansion of Nitrogen.

The thermometer reservoir and the capillary tube which forms part of it were placed in the boiling-point apparatus described on p. 72.

As regards the parts of the capillary tube included in the "dead space," these were protected from the heating effect of the boiling-point apparatus by surrounding sleeves of thin copper, traversed by a current of cold water. The temperature was observed by means of a small thermometer placed with its bulb in one of the sleeves.

The temperature of ebullition of the water was deduced from the barometric pressure, observed every 3 minutes, on the auxiliary barometer No. 3 of the Bureau, placed in a neighbouring room, the necessary corrections being of course applied.

The following example, which is one of the observations of May 13, will suffice to illustrate the course of the operations.

DETERMINATION of the 100-point.

Time.	Vernier reading.	Auxiliary thermometers of the manometer. 4365.	Auxiliary thermometers of the manometer. 4362.	Reduced barometer in millims.	Excess of pressure of steam in millims. of water.	Temperature of the dead space.
10.14 A.M.				751·79		
	1072·215	13°·835	14°·200		2·8	12°·70
17				·79		
	·220	·830	·200		2·8	·70
21				·79		
	·210	·820	·185		2·8	·80
25				·74		
	·210	·820	·190		2·9	·80
29				·72		
	·210	·825	·210		2·7	·90
32				·73		
	·225	·830	·230		2·8	·90
10.37				·72		
Means . . . . =	1072·215	13·827	14·202	751·75	2·8	12·80
Vernier correction =	+ ·004	− ·054	− ·349	+ ·25		
Index error . . =	+ 10·550					
		13·773	13·853	752·00		
	1082·769					
Corr. for dilatation =	− 2·432	Mean = 13·813				
„ „ gravity =	+ ·360					
„ „ scale . =	− ·036					
Δp . . . . . =	− ·196	T = 99·705				
	1080·465	Δv = − 21·20 microlitres				

The total volume of the "dead space" being 445·10 cub. millims., the part surrounded by the sleeves, whose volume was 74·76 cub. millims., had the temperature 12°·8 indicated in the last column on the right. For the rest of the "dead space," of volume 366·34 cub. millims., we have adopted the temperature 13°·77 indicated by the auxiliary thermometer No. 4365.

The excess of pressure of the vapour over the barometric pressure is measured by a small water manometer placed in the cork of the apparatus, and must be transformed to mercury pressure and added to the reduced barometric height. From this total pressure the temperature is deduced by means of the tables published by M. Broch for the temperatures of ebullition of pure water,* part of which is re-printed in the Appendix to this paper, Table III.

If the small variations of initial pressure during the course of the experiments be taken into account, we obtain, on applying to the observations the formulæ indicated above, the following values for the *coeffìcent of expansion of nitrogen under constant volume*:—

Date	Coefficient.		Initial pressure in millims. assumed for the calculation.
April 4	0·00367 131		
„ 5	244		
„ 7	196		
„ 8	186	0·00367 177	793·421
„ 8	134		
„ 9	172		
April 22	0·00367 173		
„ 22	117		
„ 23	148		
„ 23	167	0·00367 142	793·519
„ 25	143		
„ 25	126		
„ 26	120		
May 6	0·00367 178		
„ 6	198		
„ 7	195		
„ 7	172	0·00367 175	793·532
„ 8	140		
„ 10	145		
„ 10	194		
May 26	0·00367 246		
„ 26	224		
„ 27	227	0·00367 227	793·563
„ 27	224		
„ 28	216		
	General mean = 0·00367 180		

The general mean of these four groups of determinations has been employed for the calculation of the temperatures in the series of comparisons made about this time, excepting the series with the thermometer K.9, for which the mean of the last group of observations 0·00367 227 was adopted.

* 'Trav. et. Mém. du Bureau International,' vol. 1 A, p. 46.

XXXVI. Comparisons between Platinum Thermometer K.8 and the Nitrogen Thermometer.

These comparisons were made in the oil-bath previously described (Section XXIX). Fig. 17 shows the arrangement of the two instruments in the comparison-bath. For the first series between 88° and 116° water was employed in the jacket.

Simultaneous observations of the two instruments were made by the authors while an assistant worked the stirrer.

Each comparison at any one temperature consisted of ten observations. To eliminate slight uncertainties due to thermoelectric effects the battery current was always reversed after the first five readings.

The second series of observations, extending from 120° to 160°, was obtained by the ebullition of paraxylene, and the final series up to 190° with aniline.

As we have indicated in the résumé of the zeros on p. 82, the comparisons of K.8 with the nitrogen thermometer may be divided into two groups, the first extending from 88° to 161° (March 23 to April 2) and consisting of twenty-six observations, the second from 89° to 190° (May 14 to 24) and comprising twenty-two observations.

We give as example of a comparison the observations of May 24 at 188°·6.

Nitrogen Thermometer Readings, 24th May.

Time.	Vernier readings.		Auxiliary thermometers of the manometer. 4365 (low).	4362 (high).
	millims.			
11.25	1327·260		16°·050	17°·800
	·570		·060	·800
	·485		·070	·820
	·385		·100	·820
	·370		·100	·845
	·310		·110	·850
	·310		·110	·850
	·300		·120	·900
	·350		·130	·890
11.40	1327·350		16·140	17·880
Mean . . . . =	1327·369		16·099	17·846
Vernier correction . . . =	+ ·007	Total correction .	− ·052	− ·335
Correction for index error . =	+ 10·550		16·047	17·511
	1337·926		Mean = 16°·779	
Correction for dilatation . =	− 3·647			
„ „ gravity . . =	+ ·444			
„ „ scale . . . =	− ·048			
Δp. . . . . =	− ·346		Δv = − 24·5	
	1334·329		Nitrogen temperature = 188°·588	

PLATINUM Thermometer Readings. Experiment No. 68, 24th May, 1898.

Bridge-centre = − ·002 centim. Battery resistance = 317 ohms.
Coils B, F, J = 10488·249. Box temperature = 15°·22.
Factor = 1 + ·000 0020.

Readings.

α.		β.	
1	+14·135	6	+14·610
2	+16·875	7	+14·520
3	+16·270	8	+14·900
4	+15·330	9	+14·930
5	+14·810	10	+14·990

Mean bridge-wire reading . .	=	+ 15·137
Bridge-wire correction . . .	=	− 0·035
Corrected bridge-wire reading	=	+ 15·102
Coils	=	10488·249
Bridge-wire reading . . .	= +	15·102
Centre correction	= +	0·002
Temperature correction . .	= +	0·020
Resistance found	=	10503·373
Resistance at zero	=	6110·805
$(R - R_0)$	=	4392·568
Fundamental interval . . .	=	2361·246
Platinum temperature . . .	=	186°·027.

In reducing the observations we have taken as the mean of the determinations of initial pressure effected before and after each series the values—

		millims.
For the first series . . .	H_0 =	793·470 ;
For the second series . .	H_0 =	793·545.

XXXVII. COMPARISONS WITH THE THERMOMETER K.9.

With the thermometer K.9, which has twice the resistance of K.8, a precisely similar series of comparisons was made. The constants adopted for the calculation of the nitrogen temperatures of this series were :—

Initial pressure = 793·563 millims.
Coefficient α = 0·003 672 27,

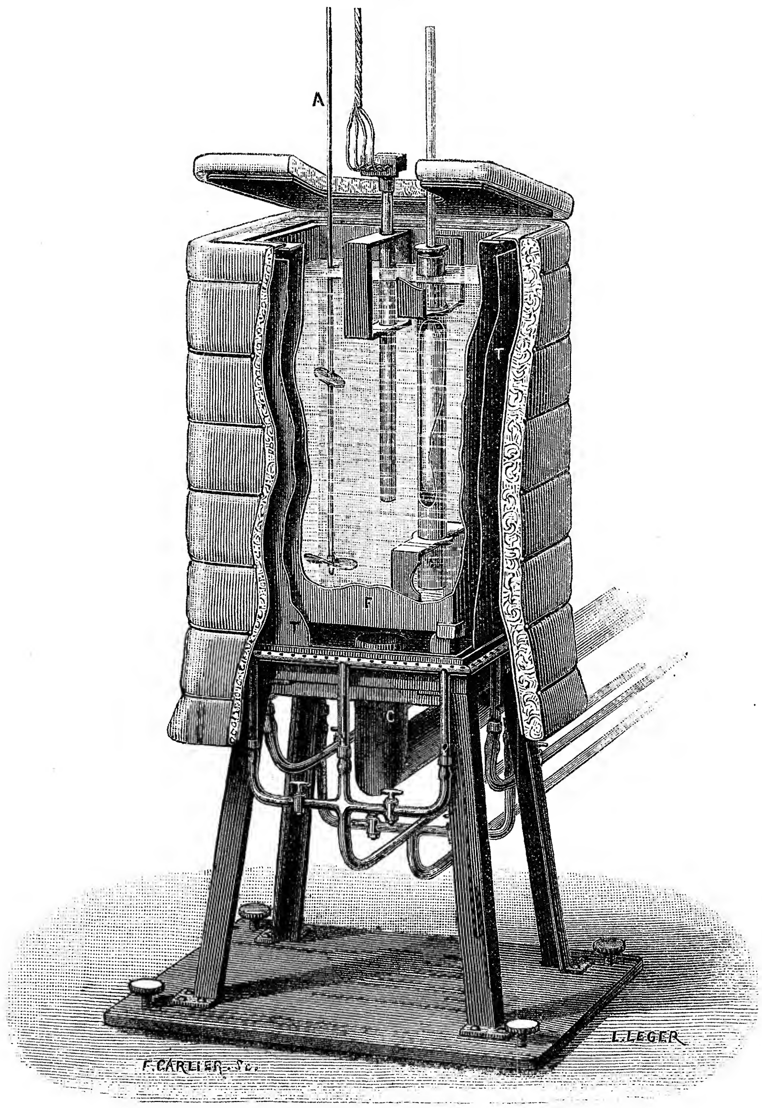
which is the mean of the last group of determinations given on p. 84.

XXXVIII. Comparisons at Temperatures between 250° and 460°.

For the comparisons at high temperatures we constructed a special heating bath, which has proved satisfactory up to the temperatures indicated, and has subsequently been employed up to about 600°.

This apparatus is represented in fig. 19. It consists essentially of a bath of a

Fig. 19.



Bath for High Temperature Comparisons.

F, cast-iron tank holding the mixed nitrates; A, stirrer shaft; C, chimney; T, wall of air-bath cut away to show interior.

mixture of nitrates of potassium and sodium, heated externally by a double circulation of hot gases, and stirred continuously by a system of rotating screws.

The cast-iron vessel which forms the bath has a depth of 50 centims. and an exterior quadrangular section of 20 centims. by 12 centims.; the angles are slightly rounded at the corners and on the bottom. The interior cross-section of the bath approaches an ellipse, a form found by one of us especially favourable to thorough stirring, when the shaft carrying the rotating screw-blades is placed at one of the foci.

The casting is supported by four substantial feet, 6 centims. in length, on a massive iron plate. This plate is pierced in the centre by a large circular hole, 8 centims. diameter, connected to a suitable sheet-iron chimney, to take away the products of combustion. Around the bath is fixed the first envelope of stout sheet-iron resting on the base-plate, whose height is a little less than that of the bath. Over this is placed a second envelope, open below, slightly pyramidal, and protected on the exterior by several layers of asbestos-card and wool. This rests on the upper edge of the bath, and may easily be detached from the rest of the apparatus.

A special rectangular burner, fitted with several gas taps, is placed round the inner envelope, near the lower opening. The hot gases rise first in the space between the two covers, then descend between the bath and the inner one, finally escaping by the chimney. The top of the bath and the whole of the hot portions of the apparatus which are exposed are prevented, as far as possible, from disturbing the temperature of the room by covering them with thick layers of asbestos-wool.

It would be dangerous to expose the thermometric reservoirs to the direct action of the melted salts. We therefore fixed in the bath thin weldless steel tubes closed at their lower ends, and projecting a few centimetres above the surface of the liquid. The thermometers were introduced into these tubes, which they fitted almost exactly. The tube containing the reservoir of the nitrogen thermometer was provided with a brass lid closely surrounding the capillary tube, a few washers of asbestos completing the joint.

The bath is stirred by two sets of screw-blades fixed to a vertical steel shaft, which extends to a height of about 40 centims. above the top. The upper end of this shaft is suspended directly by a piece of rubber tube from the axis of a small electric motor worked by four accumulators.

The system of heating which we have just described allows a very satisfactory constancy of temperature to be attained, but several hours are required in order to obtain another equilibrium at a different temperature. To facilitate this, the bath was heated continuously during the whole course of the experiments at high temperatures. For the first set of comparisons, which were interrupted by an accident, and which consisted of a small number of measurements, we employed the reservoir of "verre dur" described previously.

In the second, and more complete series, the porcelain reservoir was used throughout.

First Comparisons. (Platinum thermometers, K.8 and K.9, with "verre dur" gas thermometer.)—The comparisons numbered 70 to 72 in the table for K.8, and 25 to 32 in that for K.9 were made during the summer of 1898 with the thermometric reservoir of "verre dur." The following table shows the sequence of the various operations.

Date.	Initial pressure in millims.	Determination of the coefficient of expansion of the nitrogen.	Remarks.
June 9	...	...	Reservoir heated to 440° for four hours
,, 9	532·877		
,, 10	532·879		
,, 10	532·869		
,, 11	...	0·00366 867	
,, 11	...	846	
,, 11	...	849	
,, 12	532·905		
,, 12	532·868	844	
,, 13	...	827	
,, 17	...	...	Reservoir heated to 500°
,, 17	Commenced comparisons with thermometer K.9		
,, 19	534·320		
,, 19	534·298		
,, 19	534·307		
,, 20	534·295		
,, 20	Commenced comparisons with thermometer K.8		

The *comparisons* all being subsequent to the heating of the reservoir to 500°, the nitrogen temperatures have been calculated, assuming for the initial pressure the mean of the observations made *after* the comparisons, viz. :—

$$H = 534\cdot305 \text{ millims.}$$

It will be noticed that this value differs by 1·42 millims.* from the initial pressure observed before the comparisons given previously; this increase is obviously due to the contraction of the reservoir by the annealing effect of the high temperatures. As the reservoir had been heated to about 500° before the observations, we concluded that the contraction was produced entirely before the first measurements.

XXXIX. Determination of the Constants of the New Gas Thermometer with Porcelain Reservoir.

We have already described the porcelain reservoir and the way it is connected to the manometer tube. It remains to indicate the method by which we have measured

* This variation of pressure corresponds to 0°·7 C.

its capacity and pressure coefficient and the necessary new determinations of the "dead space."

(a) *Capacity of the Porcelain Reservoir.*

This was determined by weighing the reservoir empty, and filled with water at 0°. The following are the results of the weighings made :—

Date.	Reservoir empty.	Reservoir filled.	Weight of water.
	grams.	grams.	grams.
August 9	261·974 74		
,, 10		427·336 59	165·368 54
,, 11	261·961 35		
,, 11		427·328 86	165·367 84
,, 11	261·960 69		
,, 12		427·337 35	165·376 84
,, 12	261·960 33		
			Mean = 165·371 07

The volume occupied by this mass of water at 0° C. is

$$V_0 = 165·393 \text{ cub. centims.}$$

This value has been taken as the total capacity up to the extremity of the capillary tube, part of which is included in the "dead space."

(b) *Dilatation of the Porcelain.*

The porcelain reservoir, carefully dried, was connected to the mercury pump, very thoroughly exhausted, and then filled with mercury *in vacuo*. To the end of the capillary was cemented a glass tube about 1 millim. internal diameter, divided into millimetres, whose calibration corrections and internal volume had been previously determined. The whole being placed in melting ice, the level of the mercury was adjusted to a point on this tube, which was carefully noted. Bringing the whole afterwards to 100°, the amount of mercury which escaped was determined by weighing, the necessary corrections being applied to reduce the reading of the meniscus to its original position.

Four determinations were made by this process of the apparent dilatation of mercury in porcelain between 0° and 100°, and a few series of observations were taken at intermediate temperatures. These measurements, about which it is not necessary to enter into further detail, gave after all reductions the following results :—

Temperature.	Dilatation of the reservoir in microlitres, *i.e.*, excess over volume at 0° C.
25°·229	33·87
29·995	40·17
39·746	53·74
49·990	66·85
99·792	147·64
99·796	147·28
99·890	148·03
99·905	148·00

[For the dilatation of mercury, which enters into the calculations, the formula found by one of us ('Procès-verbaux des Séances du Comité International,' 1891, p. 37)

$$V_t = V_0[1 + (182008t - 11{\cdot}380\,4t^2 + 0{\cdot}169\,21t^3)\,10^{-9}]$$

was adopted.]

Treated by the method of least squares, these observations give for the cubic dilatation of porcelain the following formula:—

$$V_t = V_0[1 + 0{\cdot}000\,007\,593\,06t + 0{\cdot}000\,000\,013\,750t^2].$$

The observations between 0° and 100° which determine the value of the term in t^2 not being numerous, we can consider only the mean dilatation between the extreme points 0° and 100° as having been determined with sufficient accuracy. As, however, it was of importance to know the second term more exactly, as its influence increases at high temperatures, we made a second determination of the dilatation by means of the Fizeau apparatus. The specimen which served for this determination was prepared from a fragment of the capillary tube of a precisely similar reservoir made at the same time at the Imperial Porcelain Factory at Berlin.

The results of this determination, which comprised 37 observations between the temperatures 2° and 82°, are for the linear expansion

$$\alpha_1 = 0{\cdot}000\,002\,687\,62,$$

$$\beta_1 = 0{\cdot}000\,000\,002\,987\,3;$$

and for the cubical expansion

$$\alpha_2 = 0{\cdot}000\,008\,062\,8,$$

$$\beta_2 = 0{\cdot}000\,000\,008\,983.$$

The two methods give practically the same result for the mean dilatation between 0° and 100°; by the weight thermometer we have

$$\alpha_1 + 100\beta_1 = 0{\cdot}000\,008\,968\,71,$$

and by the Fizeau method we have

$$\alpha_2 + 100\beta_2 = 0{\cdot}000\ 008\ 961\ 2.$$

Admitting the coefficient β_2, deduced from the observations by the Fizeau method, we have calculated the coefficient α_2 from the relation

$$\alpha_2 + 100\beta_2 = 0{\cdot}000\ 008\ 968\ 71,$$

which gives

$$\alpha_2 = 0{\cdot}000\ 008\ 070\ 35.$$

We have thus adopted as our final formula for the cubic expansion of Berlin porcelain

$$V_t = V_0 (1 + 0{\cdot}000\ 008\ 070\ 35t + 0{\cdot}000\ 000\ 008\ 983t).$$

(c) *Pressure Coefficient of the Porcelain Reservoir.*

The measurement of the pressure coefficient of the porcelain reservoir was made in precisely the same way as that of the glass reservoir previously described on p. 75. We determined by three series of observations the variation of volume Δv corresponding to a variation of pressure of 1 millim., obtaining the following results :—

	microlitres.
(1)	$\Delta v = 0{\cdot}003\ 803\ 5$
(2)	$\Delta v = 0{\cdot}003\ 701\ 7$
(3)	$\Delta v = 0{\cdot}003\ 746\ 6$

We have adopted the mean of these three determinations, viz. :—

$$\Delta v = 0{\cdot}003\ 750 \text{ microlitre per millim.}$$

(d) *Determination of the "Dead Space."*

For this determination we followed exactly the method already described on p. 75. The nine weighings made gave divergences from the mean of four parts per thousand. After all reductions we found for the whole volume of the "dead space"

$$v = 709{\cdot}5 \text{ microlitres.}$$

The effective capacity of the thermometric reservoir being

$$V_0 = 164{\cdot}805 \text{ cub. centims.},$$

we have

$$v/V_0 = 0{\cdot}004\ 305.$$

This result, which is appreciably higher than the corresponding one for the reservoir of "verre dur," was employed for the reduction of all the measurements made with the gas thermometer with porcelain reservoir.

XL. First Determinations with Porcelain Gas Thermometer.

The mounting of the gas thermometer being completed, we proceeded to fill the reservoir with nitrogen. The gas was prepared by the process previously described and was thoroughly dried over phosphorus pentoxide. The reservoir was several times pumped out and partially filled with the dry gas, it being heated meanwhile to a temperature of about 250°, and the final filling and adjustment of the pressure was made at the same high temperature.

We give in the following table the measurements of the initial pressure and coefficient of expansion of the nitrogen, made immediately afterwards.

Date.	Initial pressure. H_0.	Coefficient. α.
	millims.	
September 18	524·591	
" 18	·619	
" 19	·576	
" 19	...	0·003 670 8
" 20	...	669 9
" 20	·592	
" 20	·589	
" 21	·577	
" 21	...	0·003 670 1
" 21	...	3 669 4
" 21	·627	
" 21	·624	
" 22	·584	
" 22	...	0·003 669 4
" 22	...	669 2
" 23	·563	
" 23	·572	
	Mean =	0·003 669 8

The value here found for the coefficient of expansion of nitrogen is slightly higher than that previously obtained with the glass reservoir thermometer (0·003 669 8 instead of 0·003 668 5).

Before proceeding to the experiments at high temperatures, we thought it advisable to heat the porcelain reservoir to the temperature of ebullition of sulphur, to see if under the actual circumstances prolonged exposure to a high temperature would produce a modification of the initial pressure. As is well known, certain bodies retain traces of water or condensed gases up to very high temperatures, and, as the reservoir had been washed with distilled water, there was some ground for apprehension that, in spite of the care taken with the filling, it might possibly have retained traces of water.

After a heating of 26 hours above 400° we found in fact an initial pressure considerably greater than that given above, viz. :—

	millims.
October 4	525·359
,, 4	525·361
,, 5	525·341
,, 7	525·348
Mean	= 525·352

Determinations of the coefficient of expansion gave also a value appreciably greater than the one found previously.

From the mean of four experiments we found

$$\alpha = 0{\cdot}003\ 674\ 0.$$

Our fears having been justified, the thermometer reservoir was again put into communication with the pump and exhausted, being meanwhile heated to a temperature of about 500°. After several successive exhaustions and partial fillings of gas the reservoir was then pumped out very thoroughly, and after remaining vacuous for 24 hours was filled to the proper pressure with very well dried gas. It was maintained all the time at a temperature approaching 500°.

The following measurements of the initial pressure and coefficient of expansion were then made :—

Date.		Initial pressure.	Coefficient.
October	15	...	0·00366 855
,,	16	...	845
,,	16	528·853	
,,	16	·824	
,,	17	·833	
,,	17	...	0·00366 830
,,	17	...	934
,,	18	528·828	
,,	18	·806	
,,	19	...	0·00366 764
,,	19	...	776
,,	20	...	769
,,	20	...	873
,,	21	528·801	
,,	22	·848	
,,	22	·844	
From October 25 to 29, comparisons with thermometer K.9.			
October	31	528·773	
November	1	·781	
,,	1	·773	
,,	2	·789	
,,	2	...	0·00366 783
,,	2	...	742
,,	3	·790	
,,	5	·734	
,,	7	528·746	
,,	10	·742	
,,	11	·746	
,,	11	·739	
From November 12 to 17, comparisons with thermometer K.8.			
November	18	528·759	
,,	18	·755	
,,	30	...	0·00366 848
,,	30	...	764
December	1	...	757
,,	1	528·765	
,,	1	·735	

From this table it may be seen that the initial pressure diminished during the series of comparisons with K.9 by about 0·05 millim. We have assumed for the reduction of the observations that this diminution was proportional to the time. During the comparisons with thermometer K.8 no sensible diminution of initial pressure was observed.

Comparisons of the Platinum Thermometers with Porcelain Gas Thermometer.

We give in the following table the values of the initial pressure assumed on each day for the experiments with thermometer K.9. For the coefficient of expansion we have taken the mean of the determinations given above, viz. :—

$$\alpha = 0{\cdot}003\ 668\ 11.$$

Date of comparison.	Initial pressure in millims.
October 25	528·807
„ 26	528·802
„ 27	528·797
„ 28	528·792
„ 29	528·787

The comparisons are numbered 33 to 53 in the table for K.9 at the end.

For the calculation of all the comparisons with thermometer K.8 we have taken as the initial pressure the value 528·747 millims. These comparisons are numbered 73 to 91 in the table for K.8 at the end.

Comparisons of Thermometer K.2.

We noticed that during the experiments with K.8 and K.9 above the sulphur point the glass tubes of the platinum thermometers were seriously attacked and showed signs of softening. Some time previously porcelain tubes had been ordered to replace the glass ones, but owing to the delivery of these being inordinately delayed, we were obliged to relinquish the comparisons we had intended to make with K.8 and K.9 at higher temperatures and take in their place a low resistance platinum thermometer, K.2, already provided with a porcelain tube. The initial pressure in the gas thermometer was reduced to 391·88 millims. and a series of 12 comparisons made, the results of which are shown in the table at the end.*

The constants of the gas thermometer were determined in the usual manner before the comparisons. The value found for the coefficient α was 0·003 667 71. We were prevented by an accident from redetermining this coefficient of dilatation *after* the measurements. This is to be regretted, as the preceding determinations showed a systematic diminution in its value which we are unable to explain.

If the coefficient corresponding to an initial pressure of 392 millims. be deduced

* As a confirmation of the general accuracy of the methods of standardization, &c., adopted in our platinum thermometry, we may mention that on the return of the apparatus from France the constants of thermometer K.2 were redetermined at Kew by Dr. CHREE and Mr. HUGO, using the improved Cambridge resistance box, which had just been re-calibrated by them. For the platinum temperature of the sulphur point at 760 millims. pressure they found a value differing only 0°·01 from that got at Sèvres, although nearly the whole of the apparatus employed, including the resistance box, leads, and barometer, were of patterns differing materially from those used in France.

from the law previously found by one of us that the departure of the coefficient from that of a perfect gas varies proportionally to the initial pressure, we have

$$\alpha = 0{\cdot}003\ 666\ 3.$$

This coefficient would give for temperatures near the sulphur point values about 0°·2 higher than those deduced by employing the one directly observed.

We have nevertheless adhered to the latter for the calculation of the temperatures of this series of comparisons, in order to avoid the introduction of any hypothesis.

XLI. Explanation of the Tables of Results.

The results of the whole of the comparisons made are given in the tables for each thermometer at the end. In these the experiments are arranged in order of ascending temperature. The first three columns give for each experiment the progressive number, the number in our note-books and the date. Columns IV. and V. give *pt* and *d*, the value for *d* being that deduced from the Callendar formula given on p. 39, assuming the value for δ as determined for each thermometer at the sulphur point, and taking our new value for the boiling-point of sulphur at 760 millims. pressure, namely 445°·27, given later on p. 101. Column VI. gives the equivalent on the nitrogen scale of the observed *pt*, as thus calculated, and Column VII. the temperature on the nitrogen scale as given by the gas thermometer. Column VIII. shows the difference between the calculated and observed values, and Column IX. the constancy of the temperature in each experiment as given by the indication of the platinum thermometer.

XLII. Determination of the Boiling-Point of Sulphur.

After ascertaining that it was possible, by means of the bath of fused nitrates, to make accurate comparisons between the platinum and gas thermometers at temperatures up to about 600°, we saw that by making alternately a determination of the resistance of a platinum thermometer at the boiling-point of sulphur, and a comparison with the gas thermometer near the same temperature, we had a means of obtaining a new determination of the boiling-point on the nitrogen scale. We accordingly made, in an apparatus of the form described by Callendar and Griffiths as the "Meyer tube," a number of determinations of the platinum temperature of sulphur-vapour boiling freely under atmospheric pressure. Readings of the barometer were taken simultaneously with those of the platinum thermometer. The reservoir of the platinum thermometer was protected from contact with any condensed sulphur which might flow down to it from the cooler part of the thermometer above, by surrounding it with an asbestos cone perforated with several holes in the base and sides to permit free circulation of the sulphur vapour within it.* It is essential for the attainment of a constant temperature that the cone should be sufficiently long to completely cover the resistance-spiral and a certain length of the stem immediately above it.

During the earlier experiments we had considerable difficulty with the sulphur tubes owing to their liability to crack on re-heating, after having once been used. We thus found it convenient, when making several consecutive sulphur point determinations, to keep the sulphur just liquid between the different sets of observations, by means of a small by-pass flame. The establishment of a constant temperature in the sulphur apparatus takes a considerable time; from half-an-hour to an hour was generally allowed after insertion of the thermometer.†

The sulphur we used was obtained from Messrs. BAIRD and TATLOCK, and was made by CHANCE'S process. Though we made no chemical tests of its purity, we have reason to believe that the impurities present, if any, exert practically no influence on the boiling-point, as a large number of determinations made at Kew showed no systematic difference in the behaviour of several different samples. Additional evidence of the purity of the sulphur used is afforded by the remarkable steadiness of the temperature of the vapour, when once the equilibrium is established.

Three independent values for the boiling-point of sulphur were obtained under different circumstances. To the first of these, obtained from the preliminary comparisons of thermometer K.9 with the gas thermometer with reservoir of "verre dur," we attach less weight than to the two subsequent ones, where K.8 and K.9 were compared with the gas thermometer fitted with the porcelain reservoir more suited for high temperatures.

We discuss the observations of the later series, taken with K.8, as an example of the method of reduction followed.

The determinations made with this thermometer of the platinum temperature of the boiling-point of sulphur were eight in number, the corresponding pressures varying from 755 to 762 millims. It is obvious that, from the experiments themselves, the platinum temperature corresponding to 760 millims. could be deduced by the method of least squares, but a formula for the variation of the boiling-point with pressure deduced from so few experiments would, however, be liable to error. We, therefore,

* This form of protector is due to HEYCOCK and NEVILLE, and is described in their paper in 'Trans. Chem. Soc.,' 1895, p. 197.

† In the use of this apparatus there are several precautions to be observed essential for good results. The liquid sulphur in the Meyer tube must extend to some few centimetres above the base plate of the apparatus. The gas burner should preferably be a large solid-flame bunsen, and the flame should be screened from draughts by asbestos-card or by a number of firebricks surrounding the apparatus. The cones are attached to the thermometer by fine iron wire. The asbestos becomes very hard on cooling, but, if, after use, the adhering sulphur is burnt off, the cones can be rendered sufficiently pliable to serve for several determinations.

attempted to collect further evidence on the subject, before proceeding to the final reduction of our results.

CALLENDAR and GRIFFITHS in re-determining the boiling-point of sulphur made no attempt to deduce any formula for the variation of this point with pressure, and, in their subsequent work, apply the one deduced by REGNAULT from his observations made in 1862.

As the results of this investigation of REGNAULT have been differently interpreted by several observers, it may be worth while here to state exactly what experiments REGNAULT made on the subject. The primary object of his work was to determine the influence of large variations of pressure on the boiling-points of a number of substances, rather than to deduce formulæ representing accurately over a limited range the variation for each substance. He made altogether eight experiments with sulphur at pressures between 250 and 3000 millims., the four nearest to the standard pressure of 760 millims. being as follows:—

Temperature on air scale.	Pressure in millims.
418°·70	467·45
440·30	679·97
447·71	763·04
485·61	1308·54

In the carrying out of these experiments REGNAULT says he had considerable difficulty, due to violent boiling and also to superheating of the vapour, especially at high pressures.

From the eight experiments REGNAULT calculated a formula for the change of temperature with pressure over the whole range; from this GRIFFITHS finds the value of dt/dp at 760 millims. to be 0·082.

It happens, however, that the experiment made at 763 millims. is one, the result of which diverges more from the calculated value than almost any other, and therefore the value to be taken as the boiling-point at 760 millims. is appreciably uncertain.

The most probable value for this point, as deduced from these observations of REGNAULT, is given by different authorities as 448°·38, 448°·34, and 447°·48.

In view of this uncertainty, and also of the fact that the Meyer tube apparatus is so entirely different in its construction from that employed by REGNAULT, we deemed it advisable to obtain some further evidence as to the validity of the application of REGNAULT's value of dt/dp to our experiments. As our own observations happened to be all made within a small pressure range, we selected from the records of the platinum thermometers in regular use at Kew Observatory, the results of the different determinations of the sulphur-point made with thermometers K.1 and K.3, and from these, calculated by least squares for each thermometer a formula representing the variation of pt with pressure, from which, by combination with the known value of $d.pt/dt$, we obtained two concordant values for dt/dp at 760 millims. The mean of these values coincided sufficiently nearly with that of REGNAULT to justify us in adopting the latter for present purposes, and the reduction of our observations to normal pressure is therefore based on the assumption of his value.*

We found that for the thermometers K.8 and K.9 the value of $d.pt/dt$, at the sulphur point, was practically identical with the mean of those previously obtained for the older thermometers, and by assuming this number and combining it with the value calculated from REGNAULT'S experiments for dt/dp, we obtained $d.pt/dp$ for K.8 and K.9.

From this the value of pt_s, the platinum temperature of the sulphur vapour at 760 millims., was then calculated for each experiment, and the mean value for each series taken.

The platinum temperatures found in these comparisons made with each thermometer near the point 445°, and their corresponding values on the nitrogen scale, were then treated by least squares, to obtain from them the nitrogen temperature equivalent to the value of pt_s found previously. Various formulæ for this calculation were tried, the most suitable one being found to be

$$T_s = x + y\,(pt - pt_s) + z\,(pt - pt_s)^2$$

where T_s is the nitrogen temperature sought corresponding to pt_s, and x, y and z are constants.†

In this calculation for thermometer K.8 were included the seven experiments numbered 85 to 91 in the table, and in the calculation weights were assigned to the individual experiments according to the constancy of the temperature. On substituting in the original equations of condition the greatest residual was found to be 0°·034, showing a satisfactory concordance between the values for T_s given by the different comparisons.

* Although we do not wish to give the formula we calculated from the observations made at Kew as the outcome of a new *determination* of dt/dp for sulphur, yet it may be worth while to give an idea of the kind of agreement between the value found and that of REGNAULT, which we adopted for the reduction of our observations. The experiments with thermometer K.1 were made between the extremes of pressure 747 and 773 millims., but the majority of them were only very slightly removed from 760 millims. The series with K.3 was better adapted for the purpose of deducing a formula, the observations being distributed fairly evenly over the range 747 to 769 millims. These two sets of experiments were made by Mr. HUGO, Senior Assistant at Kew Observatory.

If T_s be the boiling-point under 760 millims. pressure, we have for the value at 755 millims. from the formulæ deduced from REGNAULT, and from thermometers K.1 and K.3, the values $(T_s - {\cdot}41)°$, $(T_s - {\cdot}43)°$, and $(T_s - {\cdot}42)°$, respectively.

† For another method of arriving at the value of pt_s and the corresponding T_s leading to a mean result slightly different from that here given, see Appendix II., added while the paper was in press.

From the different series of experiments from which a value can be deduced by this method we have

1st Series	K.9 and glass	reservoir	T_s	= 445°·27
2nd ,,	K.9 ,, porcelain	,,	T_s	= 445·26
3rd ,,	K.8 ,, ,,	,,	T_s	= 445·29
Mean	. . . . .			= 445·27

Although we think that the extremely close agreement of these values is to some extent fortuitous, and may give an exaggerated idea of the accuracy attained in our experiments, we think that until more is known concerning the expansion at high temperatures of the material used as thermometric reservoir, 445°·27 may be taken as a close approximation to the temperature attained by the vapour of pure sulphur boiling freely under a pressure of 760 millims. *in the apparatus above described.* Whether this represents the *true* temperature, or whether the indications of the thermometer are affected to any appreciable extent by radiation and other disturbing influences, we have not attempted to consider in detail. We contented ourselves with ascertaining that the form of apparatus we used is capable of giving consistent results, and that the temperature attained in it by the vapour after the steady state has been reached really alters with the barometric pressure. We noticed that the barometer we used, and those platinum thermometers which were provided with *glass* envelopes, appeared to follow changes at very nearly the same rate. Considering that the observations of the boiling-point were only made when the barometer appeared to be fairly steady, we think that any error in the measurement of the corresponding temperatures and pressures due to difference of lag of the two instruments must have been very small.

XLIII. Reduction of Results to Normal Scale.

In view of the lack of data as to the difference between the various gas scales at high temperatures, we are unable to reduce the results of our comparisons, and the value found for the boiling-point of sulphur, to what they would have been on the scale of the hydrogen thermometer.*

* [Footnote added December 1, 1899.—From the study of the different gas scales previously made by one of us, it appears that between 0° and 100° the point of maximum difference between the hydrogen and nitrogen scales is at 40°, where the nitrogen thermometer reads higher by 0°·01. At 100° the difference between the two scales becomes zero by definition, and above that temperature it changes sign and has a value which appears not to exceed 0°·1 below 600°.

The scale of the constant volume nitrogen thermometer appears not to be independent of the initial pressure; if we may judge by the variation of the coefficient $\frac{1}{p_0}\frac{dp}{dt}$, which approaches that of hydrogen as the pressure diminishes, we may assume that the difference between the scales of the nitrogen and

Nor is it easy to apply to the results on the nitrogen scale the correction necessary to bring them to what should have been found had we been able to employ an initial pressure of 1 metre instead of 528 millims.

As has been pointed out, all our gas temperatures are referred to the *constant-volume* scale. The connection between this and the constant-pressure scale, and the corrections to be applied to each to reduce them to the absolute gas scale, have been calculated by Lord KELVIN and Dr. JOULE from their experiments on the flow of gases through porous plugs. Various formulæ giving this correction have, however, been proposed by KELVIN and JOULE themselves, and by others. In a recent paper by ROSE-INNES* a type of formula is deduced from the same observations, which applies to the results found with all the three gases experimented upon by KELVIN and JOULE. ROSE-INNES† says, "To the degree of approximation to which we are working, therefore, there is no thermodynamic correction needed for a constant-volume gas thermometer. There may be a correction involving *squares* of small quantities, which would appear on a nearer approximation. Such a correction, however, would not be worth taking into account in the case of a thermometer constructed with air or hydrogen, as the unavoidable errors of experiment would certainly be much larger than the correction."

Our result for the boiling-point of sulphur is about 0°·7 higher than that of CALLENDAR and GRIFFITHS, but it may be well to point out here that the two values are not necessarily inconsistent. The value of CALLENDAR and GRIFFITHS is given as 444°·53 for the boiling-point of sulphur *on the constant-pressrue air scale*, the air being taken under an initial pressure of 76 centims.

Our value 445°·27 we give as the equivalent of the same temperature *on the scale of the constant-volume nitrogen thermometer*, the nitrogen being taken under the initial pressure of 528 millims. It is impossible, we think, at present to say from

hydrogen thermometers varies directly as the initial pressure. Consequently, in the comparisons between 100° and 200°, where the initial pressure was about 800 millims., the difference between the two scales would be diminished to about four-fifths, and in the comparisons between 200° and 455° to about half of what it would have been with an initial pressure of one metre.

We may also remark that the coefficient of dilatation of air under constant pressure

$$\alpha = 0{\cdot}003\ 674\ 9,$$

determined by CALLENDAR and GRIFFITHS and employed by them to calculate the temperatures in their observations, is sensibly higher than that which results from the experiments of REGNAULT,

$$\alpha = 0{\cdot}003\ 670\ 0,$$

or the value obtained some time ago by one of us,

$$\alpha = 0{\cdot}003\ 670\ 8.$$

The adoption of these latter values would raise the result of CALLENDAR and GRIFFITHS about half a degree.]

* 'Phil. Mag.,' March, 1898.

† *Loc. cit.*, p. 293.

theory what difference should be found between the results in the two cases. CALLENDAR and GRIFFITHS point out, however, that the few observations they made, using their instrument as a constant-volume thermometer, gave a result for the sulphur point about half a degree higher than that found on the constant-pressure scale. If confirmed, this would account for more than half of the difference between the two results.

It may be of interest to calculate what differences there would be between temperatures expressed on CALLENDAR'S air scale and the same temperatures on the nitrogen scale based on the adoption of our value for the boiling-point of sulphur, and on the validity of the δ formula. The adoption of the new value for the sulphur boiling-point—$0^\circ{\cdot}74$ higher than that of CALLENDAR and GRIFFITHS—would raise a δ of 1·500 to 1·5423. The differences between the temperatures deduced by admitting the validity of the parabolic formula in each case are shown in the following table :—

$T_{(Call.)}$	-50°	0°	25°	50°	75°	100°	200°	400°	600°	1000°
$T_{(new)} - T_{(Call.)}$.	$+0^\circ{\cdot}03$	0°	$-0^\circ{\cdot}008$	$-0^\circ{\cdot}011$	$-0^\circ{\cdot}008$	0°	$+0^\circ{\cdot}09$	$+0^\circ{\cdot}57$	$+1^\circ{\cdot}5$	$+5^\circ{\cdot}3$.

From the results of our comparisons we might calculate a formula giving the magnitude of a small corrective term to be applied to the temperatures as deduced by the parabolic formula to reduce them to the scale of our nitrogen thermometer. This correction, however, is not the same for the different platinum thermometers we used, and an examination of the differences given in Column VIII. of the tables shows that in some places the corrective terms for the two thermometers differ by a quantity of about the same order as the corrections themselves.

Since, as we have previously explained, our own nitrogen scale is somewhat arbitrary, and its relation to the normal scale of the hydrogen thermometer is only known over a small part of the range covered by the experiments, we would suggest that, for the present, temperatures deduced by the platinum thermometer should be reduced by a parabolic formula. The results thus obtained can always be recalculated and expressed on any scale which may subsequently be adopted as the standard scale for high temperatures.

Although we found it impossible to use hydrogen at high temperatures in our gas thermometer with glass reservoir owing to some chemical action taking place between it and the glass, yet it is quite possible that a suitable material may be found for the construction of a thermometer reservoir in which this gas may be employed at high temperatures.

Until further investigations have been made as to the relations of the various gas scales at high temperatures, and as to the influence of the initial pressure and the effect of impurities and traces of water vapour in the gases employed, and until exact determinations have been made up to high temperatures of the coefficient of expansion of the material used as thermometric reservoir, we think that for the purposes of high range thermometry a scale deduced by the parabolic formula from that of the platinum thermometer will suffice. In the present state of our knowledge any attempt to improve on such a thermometric scale would be attended with such uncertainties as would probably render it futile.

XLIV. Conclusion.

In conclusion, the authors are desirous of expressing their obligation to Dr. Benoit, Director of the Bureau International des Poids et Mesures, to Professor Carey Foster, Chairman of the Kew Observatory Sub-Committee on Thermometry, and to Dr. Chree, Superintendent of the Observatory, for continued advice and help throughout the whole of their work. For the loan of several pieces of apparatus they are indebted to Professor Schuster of Manchester, and to M. Broca of the Ecole de Médecine, Paris, and for help with the calculations to M. Maudet and to Mdlles. de Bauller and Junot, Assistants at the Bureau International, and to all these they tender their sincere thanks.

TABLE I.—Summary of Experiments with Thermometer K.8. $\delta = 1{\cdot}5435$.

NOTE.—In the column headed "Observed value on gas scale," for the experiments 1–20 the gas referred to is *hydrogen*; for all subsequent experiments, *nitrogen* is intended.

I. Reference number.	II. Number in book.	III. Date of experiment.	IV. *pt.*	V. *d.*	VI. Calculated value on nitrogen scale.	VII. Observed value of gas scale.	VIII. Difference calculated–observed.	IX. Change of temperature during experiment. 1 centim. = ·04°, approximately.
			First Series.—Comparisons against mercury thermometers in alcohol bath.					
		1897						
1	20	Dec. 7	−23·897	+·448	−23·449	−23·446	−·003	First half fall 5 centims., second half constant
2	19	„ 14	−19·479	+·352	−19·127	−19·134	+·007	Small oscillations
3	18	„ 7	−10·851	+·182	−10·669	−10·672	+·003	Fall $2\frac{1}{2}$ centims.
			Second Series.—Comparisons against mercury thermometers in water bath.					
		1897						
4	17	Nov. 15	+ 2·133	−·032	2·101	+ 2·096	+·005	Rise 2 centims.
5	16	„ 8	...	...	...	...	...	Excluded on account of moisture being discovered in the thermometer
6	8	Sept. 23	5·204	−·075	5·129	+ 5·113	+·016	Very slow fall, then rise
7	15	Nov. 15	7·488	−·106	7·382	+ 7·364	+·018	Slow rise 1·5 centim.
8	6	Sept. 6	10·300	−·140	10·160	+10·144	+·016	Slow rise, then fall. Compensation
9	7	„ 6	15·050	−·196	14·854	+14·843	+·011	Rise $2\frac{1}{2}$ centims.
10	1	Aug. 24	...	...	...	...	...	Excluded on account of being done without reversing the battery current
11	3	Sept. 4	20·941	−·253	20·688	+20·676	+·012	Rise $3\frac{1}{2}$ centims.
12	2	Aug. 28	25·350	−·291	25·059	+25·050	+·009	Slow fall, then rise 4 centims.
13	12	Nov. 9	25·520	−·292	25·228	+25·217	+·011	Very small oscillations
14	13	„ 10	30·410	−·326	30·084	+30·072	+·012	Very small oscillations
15	5	Sept. 4	36·210	−·356	35·854	+35·855	−·001	Rise 4·5 centims.
16	4	„ 4	41·543	−·375	41·168	+41·172	−·004	Compensation
17	14	Nov. 11	50·425	−·386	50·038	+50·037	+·001	Very small oscillations
			Third Series.—Comparisons against mercury thermometers in oil bath.					
		1897						
18	10	Oct. 28	59·553	−·373	59·180	+59·181	−·001	Rise 2 centims.
19	11	„ 28	70·582	−·323	70·259	+70·262	−·003	Small oscillations
20	9	„ 16	78·397	−·264	78·133	+78·141	−·008	Rise 2 centims.

TABLE I. (continued).—Summary of Experiments with Thermometer K.8. $\delta = 1\cdot5435$.

NOTE.—In the column headed "Observed value on gas scale," for the experiments 1–20 the gas referred to is *hydrogen*; for all subsequent experiments, *nitrogen* is intended.

I. Reference number.	II. Number in book.	III. Date of experiment.	IV. *pt.*	V. *d.*	VI. Calculated value on nitrogen scale.	VII. Observed value on gas scale.	VIII. Difference calculated–observed.	IX. Change of temperature during experiment. 1 centim. = ·04°, approximately.
			Fourth Series.—Comparisons against nitrogen thermometer in oil bath.					
		1898						
21	23	Mar. 24	88·934	−·154	88·780	88·776	+·004	Rise 1·6 centim.
22	22	„ 24	89·120	−·152	88·968	88·961	+·007	Rise 2·1 centims.
23	60	May 21	89·880	...	...	...	...	Error made in registration of results. Cannot be traced
24	54	„ 17	89·881	−·142	89·739	89·746	−·007	Very small fall
25	41	Apr. 1	94·780	−·007	94·703	94·710	−·007	Rise 1·2 centim.
26	21	Mar. 23	99·413	−·009	99·404	99·400	+·004	Constant
27	24	„ 24	99·418	−·009	99·409	99·435	−·026	Very small changes. Compensated
28	42	Apr. 1	99·486	−·009	99·478	99·495	−·017	Constant
29	40	„ 1	99·543	−·007	99·536	99·543	−·007	Constant
30	47	May 14	99·641	−·005	99·636	99·624	+·012	Very small fall. Done with wrong battery resistance
31	48	„ 16	99·844	−·002	99·842	99·834	+·008	Constant
32	50	„ 16	99·848	−·002	99·846	99·848	−·002	Fall 1 centim.
33	53	„ 17	99·848	−·002	99·846	99·849	−·003	Very small changes
34	49	„ 16	104·904	+·082	104·986	105·000	−·014	Very small rise
35	43	Apr. 1	105·102	+·086	105·188	105·194	−·006	Very small changes
36	25	Mar. 24	108·225	+·141	108·366	108·389	−·023	Very small changes
37	28	„ 26	109·635	+·167	109·802	109·783	+·019	Very small fall
38	46	Apr. 2	110·111	+·176	110·287	110·273	+·014	Very constant
39	51	May 16	110·302	+·180	110·482	110·477	+·005	Small fall
40	44	Apr. 1	111·870	+·210	112·080	112·078	+·002	Slow rise, 1·3 centim.
41	45	„ 2	114·699	+·266	114·965	114·954	+·011	Very constant
42	52	May 16	114·786	+·267	115·053	115·039	+·014	Very constant
43	27	Mar. 25	115·365	+·281	115·646	115·638	+·008	Very small oscillations
44	26	„ 24	115·427	+·282	115·709	115·725	−·016	Small fall, then rise. Compensated

TABLE I. (continued).—Summary of Experiments with Thermometer K.8. $\delta = 1\cdot5435$.

NOTE.—In the column headed "Observed value on gas scale," for the experiments 1–20 the gas referred to is *hydrogen*; for all subsequent experiments, *nitrogen* is intended.

I. Reference number.	II. Number in book.	III. Date of experiment.	IV. *pt*.	V. *d*.	VI. Calculated value on nitrogen scale.	VII. Observed value on gas scale.	VIII. Difference calculated–observed.	IX. Change of temperature during experiment. 1 centim. = ·04°, approximately.
Fourth Series (continued).—Comparisons against nitrogen thermometer in oil bath.								
		1898						
45	31	Mar. 26	118·957	+ ·354	119·311	119·258	+·053	Rise ·8 centim.
46	37	„ 30	119·631	+ ·370	120·001	119·972	+·029	Rise 1·8 centim.
47	55	May 20	119·708	+ ·371	120·079	120·033	+·046	Very small oscillations
48	30	Mar. 26	127·540	+ ·557	128·097	128·042	+·055	Very small rise
49	56	May 20	128·968	+ ·593	129·560	129·527	+·034	Constant
50	38	Mar. 30	129·748	+ ·612	130·360	130·306	+·054	Constant
51	29	„ 26	137·315	+ ·815	138·130	138·042	+·088	Very slow rise
52	57	May 20	137·670	+ ·825	138·496	138·456	+·040	Very slow rise
53	62	„ 21	138·328	+ ·843	139·171	139·137	+·034	Very slow rise
54	39	Mar. 30	138·793	+ ·856	139·650	139·603	+·047	Constant
55	58	May 20	145·465	+1·053	146·518	146·480	+·038	Rise ·8 centim.
56	36	Mar. 30	147·706	+1·122	148·828	148·779	+·049	Very small changes. Compensated
57	34	„ 30	150·296	+1·205	151·501	151·446	+·055	Very small changes. Compensated
58	32	„ 28	152·542	+1·279	153·821	153·769	+·052	Fall 1·2 centim.
59	59	May 20	154·639	+1·351	155·990	155·940	+·050	Fall 2·5 centims.
60	61	„ 21	154·692	+1·352	156·044	156·020	+·024	Rise 1·5 centim.
61	63	„ 21	158·953	+1·499	160·452	160·429	+·023	Fall 1 centim.
62	33	Mar. 29	159·333	+1·512	160·845	160·788	+·057	Very constant
63	35	„ 30	159·891	+1·531	161·422	161·384	+·038	Very small changes. Compensated
64	64	May 23	168·019	+1·833	169·852	169·812	+·040	Rise 1 centim.
65	69	„ 24	173·654	+2·054	175·708	175·691	+·017	Very slow fall
66	67	„ 24	180·514	+2·341	182·855	182·823	+·032	Rise 5·3 centims.
67	65	„ 23	180·691	+2·346	183·037	182·989	+·048	Small changes. Compensated
68	68	„ 24	186·027	+2·581	188·608	188·608	+·000	Small changes. Compensated
69	66	„ 23	187·970	+2·670	190·640	190·624	+·016	Rise 5 centims.

TABLE I. (continued).—Summary of Experiments with Thermometer K.8. $\delta = 1{\cdot}5435$.

NOTE.—In the column headed "Observed value on gas scale," for the experiments 1–20 the gas referred to is *hydrogen*; for all subsequent experiments, *nitrogen* is intended.

I. Reference number.	II. Number in book.	III. Date of experiment.	IV. *pt.*	V. *d.*	VI. Calculated value of nitrogen scale.	VII. Observed value on gas scale.	VIII. Difference calculated–observed.	IX. Change of temperature during experiment. 1 centim. = ·04°, approximately.
Fifth Series.—Comparisons against glass reservoir gas thermometer in nitrate bath. Gas thermometer bulb in steel tube.								
		1898						
70	72	June 21	275·77	+ 8·06	283·83	283·88	− ·05	Fall 4·7 centims.
71	70	,, 20	321·10	+11·98	333·08	332·91	+ ·17	First half rise 7 centims., second half rise 1 centim.
72	71	,, 21	339·69	+13·84	353·53	353·36	+ ·17	Rise 3·6 centims., then slow fall
Sixth Series.—Comparisons against porcelain reservoir gas thermometer in nitrate bath. Both thermometers in steel tubes.								
		1898						
73	77	Nov. 14	236·839	5·31	242·14	242·10	+ ·04	Fall 6 centims.
74	78	,, 14	239·561	5·49	245·05	245·02	+ ·03	Slow fall, then rapid rise 6·7 centims.
75	80	,, 14	291·883	9·35	301·24	301·22	+ ·02	Very constant
76	79	,, 14	291·907	9·36	301·26	301·23	+ ·03	Rise 3·4 centims.
77	74	,, 12	312·259	11·16	323·42	323·28	+ ·14	Fall 1·2 centim.
78	73	,, 12	312·336	11·16	323·50	323·36	+ ·14	Very small oscillations
79	76	,, 12	333·718	13·22	346·93	346·78	+ ·15	Steady, second half rise 2·5 centims.
80	75	,, 12	334·300	13·28	347·58	347·43	+ ·15	Fall 2·2 centims.
81	82	,, 15	347·937	14·70	362·64	362·53	+ ·11	Slow fall, then rise 1·2 centim.
82	81	,, 15	348·706	14·79	363·49	363·39	+ ·10	Slow fall, then slow rise 1·5 centim.
83	84	,, 15	392·778	19·93	412·71	412·65	+ ·06	Very slow rise ·4 centim.
84	83	,, 15	393·858	20·07	413·92	414·04	− ·08	Small oscillations
85	85	,, 16	414·472	22·75	437·23	437·15	+ ·08	Large rise 9 centims.
86	86	,, 16	414·851	22·81	437·66	437·61	+ ·05	Fall 2·6 centims., then rise 2 centims.
87	88	,, 17	419·563	23·46	443·02	443·06	− ·04	Rise 3 centims.
88	89	,, 17	419·972	23·52	443·49	443·52	− ·03	Rise 7·5 centims.
89	90	,, 17	422·085	23·81	445·89	445·87	+ ·02	Very slow fall, ·5 centim.
90	91	,, 17	422·107	23·81	445·92	445·89	+ ·03	Very constant
91	87	,, 16	430·681	25·06	455·74	455·54	+ ·20	Rise 3 centims.

TABLE II.—Summary of Comparisons with Platinum Thermometer K.9. $\delta = 1{\cdot}5472$.

NOTE.—In the column headed "Observed value on gas scale," for experiments 1–6 the gas referred to is *hydrogen*; for all subsequent experiments, *nitrogen* is intended.

I. Reference number.	II. Number in book.	III. Date of experiment.	IV. *pt.*	V. *d.*	VI. Calculated value on nitrogen scale.	VII. Observed value on gas scale.	VIII. Difference calculated–observed.	IX. Change of temperature during experiment. 1 centim. = ·2°, approximately.
				First Series.—Comparisons against mercury thermometers.				
		1898						
1	19	June 3	10·242	− ·140	10·102	10·090	+ ·012	Small changes. Compensation
2	20	„ 3	20·023	− ·245	19·778	19·774	+ ·004	Rise 4·4 centims.
3	21	„ 4	30·149	− ·323	29·826	29·819	+ ·007	Small rise, then steady
4	22	„ 4	40·378	− ·371	40·007	40·004	+ ·003	Compensation
5	23	„ 4	50·278	− ·386	49·892	49·901	− ·009	Compensation
6	24	„ 4	60·203	− ·370	59·833	59·842	− ·009	Rise 2 centims. in each half
				Second Series.—Comparisons against glass reservoir gas thermometer in oil bath.				
		1898						
7	12	June 1	68·493	− ·334	68·159	68·171	− ·012	Small changes. Compensation
8	9	„ 1	80·552	− ·245	80·307	80·314	− ·007	Rise 2 centims.
9	10	„ 1	90·023	− ·140	89·883	89·901	− ·018	Rise 1·5 centim.
10	6	May 31	99·072	− ·014	99·058	99·068	− ·010	Very constant
11	11	June 1	99·158	− ·012	99·146	99·179	− ·033	Very small rise
12	7	May 31	109·006	+ ·156	109·162	109·174	− ·012	Very small rise
13	8	„ 31	116·271	+ ·300	116·571	116·577	− ·006	Very small fall
14	13	June 2						
15	14	„ 2	128·458	+ ·581	129·039	129·050	− ·011	Fall 1·5 centim.
16	18	„ 2	138·790	+ ·858	139·648	139·667	− ·019	Large fall, 8·7 centims.
17	15	„ 2	140·660	+ ·910	141·570	141·588	− ·018	Very small rise
18	16	„ 2	149·374	+ 1·177	150·551	150·591	− ·040	Rise 2 centims.
19	17	„ 2	157·548	+ 1·451	158·999	159·040	− ·041	Fall 2½ centims.
20	5	May 31	172·575	+ 2·014	174·589	174·565	+ ·024	Slight fall
21	3	„ 30	176·415	+ 2·169	178·584	178·554	+ ·030	Large rise, 8 centims.
22	4	„ 31	181·023	+ 2·365	183·388	183·387	+ ·001	Very slow fall
23	1	„ 30	181·301	+ 2·377	183·678	183·640	+ ·038	Nearly compensated
24	2	„ 30	185·179	+ 2·545	187·724	187·744	− ·020	Very constant

TABLE II. (continued).—Summary of Comparisons with Platinum Thermometer K.9. $\delta = 1{\cdot}5472$.

NOTE.—In the column headed "Observed value on gas scale," for experiments 1–6 the gas referred to is *hydrogen*; for all subsequent experiments, *nitrogen* is intended.

I. Reference number.	II. Number in book.	III. Date of experiment.	IV. *pt.*	V. *d.*	VI. Calculated value on nitrogen scale.	VII. Observed value on gas scale.	VIII. Difference calculated–observed.	IX. Change of temperature during experiment. 1 centim. = ·2°, approximately.
Third Series.—Comparisons against glass reservoir gas thermometer in nitrate bath. Gas thermometer bulb *only* in steel tube.								
		1898						
25	28	June 18	251·49	+ 6·28	257·77	257·79	– ·02	Slow fall
26	29	,, 18	297·09	+ 9·82	306·91	306·97	– ·06	Compensated
27	30	,, 18	346·77	+14·61	361·38	361·40	– ·02	Fall 4 centims.
28	31	,, 18	385·58	+19·08	404·64	404·75	– ·11	Fall 2½ centims.
29	32	,, 18	416·00	+23·02	439·02	439·13	– ·11	Fall 6 centims.
30	25	,, 17	416·61	+23·10	439·71	439·86	– ·15	Compensated
31	27	,, 17	417·32	+23·21	440·53	440·45	+ ·08	Compensated
32	26	,, 17	425·31	+24·32	449·63	449·53	+ ·10	Large fall, 9 centims.
Fourth Series.—Comparisons against porcelain reservoir gas thermometer in nitrate bath. *Both* thermometers in steel tubes.								
		1898						
33	38	Oct. 26	241·380	+ 5·617	247·00	246·94	+ ·06	Rise 4·2 centims.
34	39	,, 26	246·664	+ 5·953	252·62	252·50	+ ·12	Rise 7·8 centims.
35	40	,, 26	247·516	+ 6·012	253·53	253·43	+ ·10	Rise 7·8 centims.
36	41	,, 26	247·800	+ 6·030	253·83	253·74	+ ·09	Rise 5·7 centims.
37	42	,, 27	280·625	+ 8·458	289·08	288·97	+ ·11	Very constant
38	43	,, 27	284·328	+ 8·751	293·08	292·86	+ ·22	Rise 2 centims.
39	45	,, 27	317·138	+11·641	328·78	328·56	+ ·22	Compensated
40	44	,, 27	317·333	+11·660	328·99	328·76	+ ·23	Compensated
41	33	,, 25	318·402	+11·759	330·16	329·88	+ ·28	Large rise, 13·7 centims.
42	34	,, 25	319·484	+11·861	331·34	331·08	+ ·26	Rise 5·5 centims.
43	35	,, 25	319·523	+11·864	331·39	331·13	+ ·26	Very constant
44	46	,, 28	349·749	+14·933	364·68	364·43	+ ·25	Very constant
45	47	,, 28	349·764	+14·936	364·70	364·46	+ ·24	Slight fall
46	49	,, 28	386·868	+19·241	406·11	405·92	+ ·19	Slow rise, 2 centims.
47	48	,, 28	386·999	+19·256	406·26	406·09	+ ·17	Fall 6·4 centims.
48	36	,, 25	397·090	+20·523	417·61	417·28	+ ·33	Very small rise
49	37	,, 25	398·380	+20·657	419·04	418·70	+ ·34	Fast rise, 10 centims.
50	52	,, 29	408·348	+21·986	430·33	430·25	+ ·08	Fall 5 centims.
51	53	,, 29	408·587	+22·020	430·61	430·49	+ ·12	Rise 4·7 centims.
52	50	,, 29	424·922	+24·262	449·18	449·12	+ ·06	Very slight fall
53	51	,, 29	426·258	+24·447	450·70	450·58	+ ·12	Constant

TABLE III.—Summary of Observations with Platinum Thermometer K.2. $\delta = 1{\cdot}554$.

I. Reference number.	II. Number in book.	III. Date.	IV. *pt*.	V. *d*.	VI. T calculated on nitrogen scale.	VII. T observed on nitrogen scale.	VIII. Calculated–observed.	IX. Constancy of temperature in the experiment. 1 centim. = ·2°, approximately.
		1898						
1	4	Dec. 16	403·11	+21·41	424·52	424·27	+·25	Steady, then rise of ·8 centim.
2	5	„ 16	430·62	+25·20	455·82	455·60	+·22	Fall ·5 centim., then steady
3	10	„ 16	431·25	+25·29	456·54	456·36	+·18	Fall in first half 1 centim., in second ·7 centim.
4	9	„ 13	437·34	+26·18	463·52	463·17	+·35	Rise 2·3 centim.
5	7	„ 13	437·72	+26·24	463·96	463·52	+·44	Constant
6	8	„ 15	447·48	+27·70	475·18	475·04	+·14	Slow rise
7	11	„ 14	489·20	+34·48	523·68	523·42	+·26	Fall ·4 centim., then rise ·9 centim.
8	12	„ 14	490·55	+34·71	525·26	525·05	+·21	Fall 1·4 centim.
9	6	„ 13	493·77	+35·26	529·03	528·62	+·41	Fall ·2 centim., then rise 1 centim.
10	3	„ 13	494·64	+35·42	530·06	529·68	+·38	Rise 2·3 centims., then fall. Compensated
11	2	„ 14	541·92	+44·29	586·21	586·05	+·16	Fall 1·6 centim.
12	1	„ 14	542·23	+44·35	586·58	586·46	+·12	Rise 2·5 centims., then fall ·5 centim.

APPENDIX I.

With a view to facilitating the calculations involved in platinum thermometry, we give, as an appendix, several tables calculated by Mdlle. DE BAULLER and M. MAUDET, of the Bureau International, which have proved of great utility during our work.

Table I. gives from $-40°$ to $460°$ the values of $\left[\left(\frac{T}{100}\right)^2 - \frac{T}{100}\right]$, and the product of this quantity into a number of different values of δ. It is used for deducing pt from given values of T.

Table II. is for the resolution of the converse problem, and gives T corresponding to different values of pt for thermometers having a δ between 1·54 and 1·57. At the side are given differences for interpolation between the whole degrees.

Table III. is for the reduction of the steam points, and is extracted from the table calculated by M. BROCH from the results of REGNAULT for the boiling-point of water under different pressures.

Table IV. is for reducing to its equivalent in mercury pressure the excess of pressure of the steam in a boiling-point determination, as measured in millimetres of water.

Table V. is for converting to the platinum scale the temperature of the steam as obtained from Table III.

As an example of the use of some of these tables, we give the following:—

Let the resistance in ice of a certain platinum thermometer whose $\delta = 1·500$ be 2·57827 ohms, and let its resistance in steam be 3·57298 ohms, the barometric height at the time, corrected for temperature and reduced to sea level and latitude 45°, being 749·96 millims., and the excess of the steam pressure over that of the atmosphere being 1·8 millim. of water. Find the resistance corresponding to 100°.

From Table IV. the mercury pressure corresponding to 1·8 millim. of water = 0·13 millim. Adding this to the barometric height we obtain 750·09 millims. as the total pressure of the steam. Then from Table III. we obtain, as the temperature of the steam at this pressure, 99°·6343.

For $\delta = 1·500$ we find from Table V. that the platinum temperature corresponding to 99°·6343 is 99°·6343 + 0°·0055 = 99°·6398.

		Ohms.
Therefore the change of resistance between the platinum temperatures 0° and 99°·6398 is .	(3·57298 − 2·57827)	= 0·99471
And so F.I. (the fundamental interval, *i.e.*, the change between 0° and 100°)	$= \frac{0·99471 \times 100}{99·6398}$	= 0·99831
Therefore, resistance at 100°		= 3·57658

We append also the various formulæ showing the relations of each of the four quantities T, *pt*, *d*, and δ to the others, viz.:—

$$d \equiv \mathrm{T} - pt = \delta\left[\left(\frac{\mathrm{T}}{100}\right)^2 - \frac{\mathrm{T}}{100}\right] = \left(\frac{5000}{\delta} + 50\right) - pt - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}},$$

$$\mathrm{T} = pt + d = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}},$$

$$pt = \mathrm{T} - d = \mathrm{T} - \delta\left[\left(\frac{\mathrm{T}}{100}\right)^2 - \frac{\mathrm{T}}{100}\right].$$

APPENDIX TABLE I.—To find d for given values of T and δ. Deduced from the formula $d = \delta\left[\left(\frac{T}{100}\right)^2 - \frac{T}{100}\right]$.

T.		$\left(\frac{t}{100}\right)^2 - \frac{t}{100}$.	δ = 1·500.	δ = 1·510.	δ = 1·520.	δ = 1·530.	δ = 1·540.	δ = 1·550.	δ = 1·560.	δ = 1·570.	δ = 1·580.	δ = 1·590.	δ = 1·600.
	−40	+0·5600	+0·8400	+0·8456	+0·8512	+0·8568	+0·8624	+0·8680	+0·8736	+0·8792	+0·8848	+0·8904	+0·8960
	−35	+0·4725	+0·7087	+0·7134	+0·7182	+0·7229	+0·7276	+0·7324	+0·7371	+0·7418	+0·7465	+0·7513	+0·7560
	−30	+0·3900	+0·5850	+0·5889	+0·5928	+0·5967	+0·6006	+0·6045	+0·6084	+0·6123	+0·6162	+0·6201	+0·6240
	−25	+0·3125	+0·4687	+0·4719	+0·4750	+0·4781	+0·4812	+0·4844	+0·4875	+0·4906	+0·4937	+0·4969	+0·5000
	−20	+0·2400	+0·3600	+0·3624	+0·3648	+0·3672	+0·3696	+0·3720	+0·3744	+0·3768	+0·3792	+0·3816	+0·3840
	−15	+0·1725	+0·2587	+0·2605	+0·2622	+0·2639	+0·2656	+0·2674	+0·2691	+0·2708	+0·2725	+0·2743	+0·2760
	−10	+0·1100	+0·1650	+0·1661	+0·1672	+0·1683	+0·1694	+0·1705	+0·1716	+0·1727	+0·1738	+0·1749	+0·1760
	−5	+0·0525	+0·0787	+0·0793	+0·0798	+0·0803	+0·0808	+0·0814	+0·0819	+0·0824	+0·0829	+0·0835	+0·0840
0	100	−0·0000	−0·0000	−0·0000	−0·0000	−0·0000	−0·0000	−0·0000	−0·0000	−0·0000	−0·0000	−0·0000	−0·0000
1	99	−0·0099	−0·0148	−0·0149	−0·0150	−0·0151	−0·0152	−0·0153	−0·0154	−0·0155	−0·0156	−0·0157	−0·0158
2	98	−0·0196	−0·0294	−0·0296	−0·0298	−0·0300	−0·0302	−0·0304	−0·0306	−0·0308	−0·0310	−0·0312	−0·0314
3	97	−0·0291	−0·0436	−0·0439	−0·0442	−0·0445	−0·0448	−0·0451	−0·0454	−0·0457	−0·0460	−0·0463	−0·0466
4	96	−0·0384	−0·0576	−0·0580	−0·0584	−0·0588	−0·0591	−0·0595	−0·0599	−0·0603	−0·0607	−0·0611	−0·0614
5	95	−0·0475	−0·0712	−0·0717	−0·0722	−0·0727	−0·0731	−0·0736	−0·0741	−0·0746	−0·0750	−0·0755	−0·0760
6	94	−0·0564	−0·0846	−0·0852	−0·0857	−0·0863	−0·0869	−0·0874	−0·0880	−0·0885	−0·0891	−0·0897	−0·0902
7	93	−0·0651	−0·0976	−0·0983	−0·0990	−0·0996	−0·1003	−0·1009	−0·1016	−0·1022	−0·1029	−0·1035	−0·1042
8	92	−0·0736	−0·1104	−0·1111	−0·1119	−0·1126	−0·1133	−0·1142	−0·1148	−0·1156	−0·1163	−0·1170	−0·1178
9	91	−0·0819	−0·1228	−0·1237	−0·1245	−0·1253	−0·1261	−0·1269	−0·1278	−0·1286	−0·1294	−0·1302	−0·1310
10	90	−0·0900	−0·1350	−0·1359	−0·1368	−0·1377	−0·1386	−0·1395	−0·1404	−0·1413	−0·1422	−0·1431	−0·1440
11	89	−0·0979	−0·1468	−0·1478	−0·1488	−0·1498	−0·1508	−0·1517	−0·1527	−0·1537	−0·1547	−0·1557	−0·1566
12	88	−0·1056	−0·1584	−0·1595	−0·1605	−0·1616	−0·1626	−0·1637	−0·1647	−0·1658	−0·1668	−0·1679	−0·1690
13	87	−0·1131	−0·1696	−0·1708	−0·1719	−0·1730	−0·1742	−0·1753	−0·1764	−0·1776	−0·1787	−0·1798	−0·1810
14	86	−0·1204	−0·1806	−0·1818	−0·1830	−0·1842	−0·1854	−0·1866	−0·1878	−0·1890	−0·1902	−0·1914	−0·1926
15	85	−0·1275	−0·1912	−0·1925	−0·1938	−0·1951	−0·1963	−0·1976	−0·1989	−0·2002	−0·2014	−0·2027	−0·2040
16	84	−0·1344	−0·2016	−0·2029	−0·2043	−0·2056	−0·2070	−0·2083	−0·2097	−0·2110	−0·2124	−0·2137	−0·2150
17	83	−0·1411	−0·2116	−0·2131	−0·2145	−0·2159	−0·2173	−0·2187	−0·2201	−0·2215	−0·2229	−0·2243	−0·2258
18	82	−0·1476	−0·2214	−0·2229	−0·2244	−0·2258	−0·2273	−0·2288	−0·2303	−0·2317	−0·2332	−0·2347	−0·2362
19	81	−0·1539	−0·2308	−0·2324	−0·2339	−0·2355	−0·2370	−0·2385	−0·2401	−0·2416	−0·2432	−0·2447	−0·2462
20	80	−0·1600	−0·2400	−0·2416	−0·2432	−0·2448	−0·2464	−0·2480	−0·2496	−0·2512	−0·2528	−0·2544	−0·2560
21	79	−0·1659	−0·2488	−0·2505	−0·2522	−0·2538	−0·2555	−0·2571	−0·2588	−0·2605	−0·2621	−0·2638	−0·2654
22	78	−0·1716	−0·2574	−0·2591	−0·2608	−0·2625	−0·2643	−0·2660	−0·2677	−0·2694	−0·2711	−0·2728	−0·2746
23	77	−0·1771	−0·2656	−0·2674	−0·2692	−0·2710	−0·2727	−0·2745	−0·2763	−0·2780	−0·2798	−0·2816	−0·2834
24	76	−0·1824	−0·2736	−0·2754	−0·2772	−0·2791	−0·2809	−0·2827	−0·2845	−0·2864	−0·2882	−0·2900	−0·2918
25	75	−0·1875	−0·2812	−0·2831	−0·2850	−0·2869	−0·2887	−0·2906	−0·2925	−0·2944	−0·2962	−0·2981	−0·3000
26	74	−0·1924	−0·2886	−0·2905	−0·2924	−0·2944	−0·2963	−0·2982	−0·3001	−0·3021	−0·3040	−0·3059	−0·3078

APPENDIX TABLE I. (continued).—To find d for given values of T and δ. Deduced from the formula $d = \delta\left[\left(\frac{T}{100}\right)^2 - \frac{T}{100}\right]$.

T.		$\left(\frac{t}{100}\right)^2 - \frac{t}{100}$.	$\delta = 1.500$.	$\delta = 1.510$.	$\delta = 1.520$.	$\delta = 1.530$.	$\delta = 1.540$.	$\delta = 1.550$.	$\delta = 1.560$.	$\delta = 1.570$.	$\delta = 1.580$.	$\delta = 1.590$.	$\delta = 1.600$.
27	73	−0·1971	−0·2956	−0·2976	−0·2996	−0·3016	−0·3035	−0·3055	−0·3075	−0·3094	−0·3114	−0·3134	−0·3154
28	72	−0·2016	−0·3024	−0·3044	−0·3064	−0·3084	−0·3105	−0·3125	−0·3145	−0·3165	−0·3185	−0·3205	−0·3226
29	71	−0·2059	−0·3088	−0·3109	−0·3130	−0·3150	−0·3171	−0·3191	−0·3212	−0·3233	−0·3253	−0·3274	−0·3294
30	70	−0·2100	−0·3150	−0·3171	−0·3192	−0·3213	−0·3234	−0·3255	−0·3276	−0·3297	−0·3318	−0·3339	−0·3360
31	69	−0·2139	−0·3208	−0·3230	−0·3251	−0·3273	−0·3294	−0·3315	−0·3337	−0·3358	−0·3380	−0·3401	−0·3422
32	68	−0·2176	−0·3264	−0·3286	−0·3308	−0·3329	−0·3351	−0·3373	−0·3395	−0·3416	−0·3438	−0·3460	−0·3482
33	67	−0·2211	−0·3316	−0·3339	−0·3361	−0·3383	−0·3405	−0·3427	−0·3449	−0·3471	−0·3493	−0·3515	−0·3538
34	66	−0·2244	−0·3366	−0·3388	−0·3411	−0·3433	−0·3456	−0·3478	−0·3501	−0·3523	−0·3546	−0·3568	−0·3590
35	65	−0·2275	−0·3412	−0·3435	−0·3458	−0·3481	−0·3503	−0·3526	−0·3549	−0·3572	−0·3594	−0·3617	−0·3640
36	64	−0·2304	−0·3456	−0·3479	−0·3502	−0·3525	−0·3548	−0·3571	−0·3594	−0·3617	−0·3640	−0·3663	−0·3686
37	63	−0·2331	−0·3496	−0·3520	−0·3543	−0·3566	−0·3590	−0·3613	−0·3636	−0·3660	−0·3683	−0·3706	−0·3730
38	62	−0·2356	−0·3534	−0·3558	−0·3581	−0·3605	−0·3628	−0·3652	−0·3675	−0·3699	−0·3722	−0·3746	−0·3770
39	61	−0·2379	−0·3568	−0·3592	−0·3616	−0·3640	−0·3664	−0·3687	−0·3711	−0·3735	−0·3759	−0·3783	−0·3806
40	60	−0·2400	−0·3600	−0·3624	−0·3648	−0·3672	−0·3696	−0·3720	−0·3744	−0·3768	−0·3792	−0·3816	−0·3840
41	59	−0·2419	−0·3628	−0·3653	−0·3677	−0·3701	−0·3725	−0·3749	−0·3774	−0·3798	−0·3822	−0·3846	−0·3870
42	58	−0·2436	−0·3654	−0·3678	−0·3703	−0·3727	−0·3751	−0·3776	−0·3800	−0·3825	−0·3849	−0·3873	−0·3898
43	57	−0·2451	−0·3676	−0·3701	−0·3726	−0·3750	−0·3775	−0·3799	−0·3824	−0·3848	−0·3873	−0·3897	−0·3922
44	56	−0·2464	−0·3696	−0·3721	−0·3745	−0·3770	−0·3795	−0·3819	−0·3844	−0·3868	−0·3893	−0·3918	−0·3942
45	55	−0·2475	−0·3712	−0·3737	−0·3762	−0·3787	−0·3811	−0·3836	−0·3861	−0·3886	−0·3910	−0·3935	−0·3960
46	54	−0·2484	−0·3726	−0·3751	−0·3776	−0·3801	−0·3825	−0·3850	−0·3875	−0·3900	−0·3925	−0·3950	−0·3974
47	53	−0·2491	−0·3736	−0·3761	−0·3786	−0·3811	−0·3836	−0·3861	−0·3886	−0·3911	−0·3936	−0·3961	−0·3986
48	52	−0·2496	−0·3744	−0·3769	−0·3794	−0·3819	−0·3844	−0·3869	−0·3894	−0·3919	−0·3944	−0·3969	−0·3994
49	51	−0·2499	−0·3748	−0·3773	−0·3798	−0·3823	−0·3848	−0·3873	−0·3898	−0·3923	−0·3948	−0·3973	−0·3998
50	50	−0·2500	−0·3750	−0·3775	−0·3800	−0·3825	−0·3850	−0·3875	−0·3900	−0·3925	−0·3950	−0·3975	−0·4000
+105		+0·0525	+0·0787	+0·0793	+0·0798	+0·0803	+0·0808	+0·0814	+0·0819	+0·0824	+0·0829	+0·0835	+0·0840
+110		+0·1100	+0·1650	+0·1661	+0·1672	+0·1683	+0·1694	+0·1705	+0·1716	+0·1727	+0·1738	+0·1749	+0·1760
+115		+0·1725	+0·2587	+0·2605	+0·2622	+0·2639	+0·2656	+0·2674	+0·2691	+0·2708	+0·2725	+0·2743	+0·2760
+120		+0·2400	+0·3600	+0·3624	+0·3648	+0·3672	+0·3696	+0·3720	+0·3744	+0·3768	+0·3792	+0·3816	+0·3840
+125		+0·3125	+0·4687	+0·4719	+0·4750	+0·4781	+0·4812	+0·4844	+0·4875	+0·4906	+0·4937	+0·4969	+0·5000
+130		+0·3900	+0·5850	+0·5889	+0·5928	+0·5967	+0·6006	+0·6045	+0·6084	+0·6123	+0·6162	+0·6201	+0·6240
+135		+0·4725	+0·7087	+0·7134	+0·7182	+0·7229	+0·7276	+0·7324	+0·7371	+0·7418	+0·7465	+0·7513	+0·7560
+140		+0·5600	+0·8400	+0·8456	+0·8512	+0·8568	+0·8624	+0·8680	+0·8736	+0·8792	+0·8848	+0·8904	+0·8960
+145		+0·6525	+0·9787	+0·9853	+0·9918	+0·9983	+1·0048	+1·0114	+1·0179	+1·0244	+1·0309	+1·0375	+1·0440
+150		+0·7500	+1·1250	+1·1325	+1·1400	+1·1470	+1·1550	+1·1625	+1·1700	+1·1775	+1·1850	+1·1925	+1·2000
+155		+0·8525	+1·2787	+1·2873	+1·2958	+1·3043	+1·3128	+1·3214	+1·3299	+1·3384	+1·3469	+1·3555	+1·3640

Appendix Table I. (continued).—To find d for given values of T and δ. Deduced from the formula $d = \delta\left[\left(\frac{T}{100}\right)^2 - \frac{T}{100}\right]$.

T.	$\left(\frac{t}{100}\right)^2 - \frac{t}{100}$.	δ = 1·500.	δ = 1·510.	δ = 1·520.	δ = 1·530.	δ = 1·540.	δ = 1·550.	δ = 1·560.	δ = 1·570.	δ = 1·580.	δ = 1·590.	δ = 1·600.
+160	+0·9600	+ 1·4400	+ 1·4496	+ 1·4592	+ 1·4688	+ 1·4784	+ 1·4880	+ 1·4976	+ 1·5072	+ 1·5168	+ 1·5264	+ 1·5360
+165	+1·0725	+ 1·6087	+ 1·6195	+ 1·6302	+ 1·6409	+ 1·6516	+ 1·6624	+ 1·6731	+ 1·6838	+ 1·6945	+ 1·7053	+ 1·7160
+170	+1·1900	+ 1·7850	+ 1·7969	+ 1·8088	+ 1·8207	+ 1·8326	+ 1·8445	+ 1·8564	+ 1·8683	+ 1·8802	+ 1·8921	+ 1·9040
+175	+1·3125	+ 1·9687	+ 1·9819	+ 1·9950	+ 2·0081	+ 2·0212	+ 2·0344	+ 2·0475	+ 2·0606	+ 2·0737	+ 2·0869	+ 2·1000
+180	+1·4400	+ 2·1600	+ 2·1744	+ 2·1888	+ 2·2032	+ 2·2176	+ 2·2320	+ 2·2464	+ 2·2608	+ 2·2752	+ 2·2896	+ 2·3040
+185	+1·5725	+ 2·3587	+ 2·3745	+ 2·3902	+ 2·4059	+ 2·4216	+ 2·4374	+ 2·4531	+ 2·4688	+ 2·4845	+ 2·5003	+ 2·5160
+190	+1·7100	+ 2·5650	+ 2·5821	+ 2·5992	+ 2·6163	+ 2·6334	+ 2·6505	+ 2·6676	+ 2·6847	+ 2·7018	+ 2·7189	+ 2·7360
+195	+1·8525	+ 2·7787	+ 2·7973	+ 2·8158	+ 2·8343	+ 2·8528	+ 2·8714	+ 2·8899	+ 2·9084	+ 2·9269	+ 2·9455	+ 2·9640
+200	+2·0000	+ 3·0000	+ 3·0200	+ 3·0400	+ 3·0600	+ 3·0800	+ 3·1000	+ 3·1200	+ 3·1400	+ 3·1600	+ 3·1800	+ 3·2000
+205	+2·1525	+ 3·2287	+ 3·2503	+ 3·2718	+ 3·2933	+ 3·3148	+ 3·3364	+ 3·3579	+ 3·3794	+ 3·4009	+ 3·4225	+ 3·4440
+210	+2·3100	+ 3·4650	+ 3·4881	+ 3·5112	+ 3·5343	+ 3·5574	+ 3·5805	+ 3·6036	+ 3·6267	+ 3·6498	+ 3·6729	+ 3·6960
+215	+2·4725	+ 3·7087	+ 3·7335	+ 3·7582	+ 3·7829	+ 3·8076	+ 3·8324	+ 3·8571	+ 3·8818	+ 3·9065	+ 3·9313	+ 3·9560
+220	+2·6400	+ 3·9600	+ 3·9864	+ 4·0128	+ 4·0392	+ 4·0656	+ 4·0920	+ 4·1184	+ 4·1448	+ 4·1712	+ 4·1976	+ 4·2240
+225	+2·8125	+ 4·2187	+ 4·2469	+ 4·2750	+ 4·3031	+ 4·3312	+ 4·3594	+ 4·3875	+ 4·4156	+ 4·4437	+ 4·4719	+ 4·5000
+230	+2·9900	+ 4·4850	+ 4·5149	+ 4·5448	+ 4·5747	+ 4·6046	+ 4·6345	+ 4·6644	+ 4·6943	+ 4·7242	+ 4·7541	+ 4·7840
+235	+3·1725	+ 4·7587	+ 4·7905	+ 4·8222	+ 4·8539	+ 4·8856	+ 4·9174	+ 4·9491	+ 4·9808	+ 5·0125	+ 5·0443	+ 5·0760
+240	+3·3600	+ 5·0400	+ 5·0736	+ 5·1072	+ 5·1408	+ 5·1744	+ 5·2080	+ 5·2416	+ 5·2752	+ 5·3088	+ 5·3424	+ 5·3760
+245	+3·5525	+ 5·3287	+ 5·3643	+ 5·3998	+ 5·4353	+ 5·4708	+ 5·5064	+ 5·5419	+ 5·5774	+ 5·6129	+ 5·6485	+ 5·6840
+250	+3·7500	+ 5·6250	+ 5·6625	+ 5·7000	+ 5·7375	+ 5·7750	+ 5·8125	+ 5·8500	+ 5·8875	+ 5·9250	+ 5·9625	+ 6·0000
+255	+3·9525	+ 5·9287	+ 5·9683	+ 6·0078	+ 6·0473	+ 6·0868	+ 6·1264	+ 6·1659	+ 6·2054	+ 6·2449	+ 6·2845	+ 6·3240
+260	+4·1600	+ 6·2400	+ 6·2816	+ 6·3232	+ 6·3648	+ 6·4064	+ 6·4480	+ 6·4896	+ 6·5312	+ 6·5728	+ 6·6144	+ 6·6560
+265	+4·3725	+ 6·5587	+ 6·6025	+ 6·6462	+ 6·6899	+ 6·7336	+ 6·7774	+ 6·8211	+ 6·8648	+ 6·9085	+ 6·9523	+ 6·9960
+270	+4·5900	+ 6·8850	+ 6·9309	+ 6·9768	+ 7·0227	+ 7·0686	+ 7·1145	+ 7·1604	+ 7·2063	+ 7·2522	+ 7·2981	+ 7·3440
+275	+4·8125	+ 7·2187	+ 7·2669	+ 7·3150	+ 7·3631	+ 7·4112	+ 7·4594	+ 7·5075	+ 7·5556	+ 7·6037	+ 7·6519	+ 7·7000
+280	+5·0400	+ 7·5600	+ 7·6104	+ 7·6608	+ 7·7112	+ 7·7616	+ 7·8120	+ 7·8624	+ 7·9128	+ 7·9632	+ 8·0136	+ 8·0640
+285	+5·2725	+ 7·9087	+ 7·9615	+ 8·0142	+ 8·0669	+ 8·1196	+ 8·1724	+ 8·2251	+ 8·2778	+ 8·3305	+ 8·3833	+ 8·4360
+290	+5·5100	+ 8·2650	+ 8·3201	+ 8·3752	+ 8·4303	+ 8·4854	+ 8·5405	+ 8·5956	+ 8·6507	+ 8·7058	+ 8·7609	+ 8·8160
+295	+5·7525	+ 8·6287	+ 8·6863	+ 8·7438	+ 8·8013	+ 8·8588	+ 8·9164	+ 8·9739	+ 9·0314	+ 9·0889	+ 9·1465	+ 9·2040
+300	+6·0000	+ 9·0000	+ 9·0600	+ 9·1200	+ 9·1800	+ 9·2400	+ 9·3000	+ 9·3600	+ 9·4200	+ 9·4800	+ 9·5400	+ 9·6000
+305	+6·2525	+ 9·3787	+ 9·4413	+ 9·5038	+ 9·5663	+ 9·6288	+ 9·6914	+ 9·7539	+ 9·8164	+ 9·8789	+ 9·9415	+10·0040
+310	+6·5100	+ 9·7650	+ 9·8301	+ 9·8952	+ 9·9603	+10·0254	+10·0905	+10·1556	+10·2207	+10·2858	+10·3509	+10·4160
+315	+6·7725	+10·1587	+10·2265	+10·2942	+10·3619	+10·4296	+10·4974	+10·5651	+10·6328	+10·7005	+10·7683	+10·8360
+320	+7·0400	+10·5600	+10·6304	+10·7008	+10·7712	+10·8415	+10·9120	+10·9824	+11·0528	+11·1232	+11·1936	+11·2640
+325	+7·3125	+10·9687	+11·0419	+11·1150	+11·1881	+11·2612	+11·3344	+11·4075	+11·4806	+11·5537	+11·6269	+11·7000
+330	+7·5900	+11·3850	+11·4609	+11·5368	+11·6127	+11·6886	+11·7645	+11·8404	+11·9163	+11·9922	+12·0681	+12·1440

APPENDIX TABLE I. (continued).—To find d for given values of T and δ. Deduced from the formula $d = \delta\left[\left(\frac{T}{100}\right)^2 - \frac{T}{100}\right]$.

T.	$\left(\frac{t}{100}\right)^2 - \frac{t}{100}$.	δ = 1·500.	δ = 1·510.	δ = 1·520.	δ = 1·530.	δ = 1·540.	δ = 1·550.	δ = 1·560.	δ = 1·570.	δ = 1·580.	δ = 1·590.	δ = 1·600.
+335	+ 7·8725	+11·8087	+11·8875	+11·9662	+12·0449	+12·1236	+12·2024	+12·2811	+12·3598	+12·4385	+12·5173	+12·5960
+340	+ 8·1600	+12·2400	+12·3216	+12·4032	+12·4848	+12·5664	+12·6480	+12·7296	+12·8112	+12·8928	+12·9744	+13·0560
+345	+ 8·4525	+12·6787	+12·7633	+12·8478	+12·9323	+13·0168	+13·1014	+13·1859	+13·2704	+13·3549	+13·4395	+13·5240
+350	+ 8·7500	+13·1250	+13·2125	+13·3000	+13·3875	+13·4750	+13·5625	+13·6500	+13·7375	+13·8250	+13·9125	+14·0000
+355	+ 9·0525	+13·5787	+13·6693	+13·7598	+13·8503	+13·9408	+14·0314	+14·1219	+14·2124	+14·3029	+14·3935	+14·4840
+360	+ 9·3600	+14·0400	+14·1336	+14·2272	+14·3208	+14·4144	+14·5080	+14·6016	+14·6952	+14·7888	+14·8824	+14·9760
+365	+ 9·6725	+14·5087	+14·6055	+14·7022	+14·7989	+14·8956	+14·9924	+15·0891	+15·1858	+15·2825	+15·3793	+15·4760
+370	+ 9·9900	+14·9850	+15·0849	+15·1848	+15·2847	+15·3846	+15·4845	+15·5844	+15·6843	+15·7842	+15·8841	+15·9840
+375	+10·3125	+15·4687	+15·5719	+15·6750	+15·7781	+15·8812	+15·9844	+16·0875	+16·1906	+16·2937	+16·3969	+16·5000
+380	+10·6400	+15·9600	+16·0664	+16·1728	+16·2792	+16·3856	+16·4920	+16·5984	+16·7048	+16·8112	+16·9176	+17·0240
+385	+10·9725	+16·4587	+16·5685	+16·6782	+16·7879	+16·8976	+17·0074	+17·1171	+17·2268	+17·3365	+17·4463	+17·5560
+390	+11·3100	+16·9650	+17·0781	+17·1912	+17·3043	+17·4174	+17·5305	+17·6436	+17·7567	+17·8698	+17·9829	+18·0960
+395	+11·6525	+17·4787	+17·5953	+17·7118	+17·8283	+17·9448	+18·0614	+18·1779	+18·2944	+18·4109	+18·5275	+18·6440
+400	+12·0000	+18·0000	+18·1200	+18·2400	+18·3600	+18·4800	+18·6000	+18·7200	+18·8400	+18·9600	+19·0800	+19·2000
+405	+12·3525	+18·5287	+18·6523	+18·7758	+18·8993	+19·0228	+19·1464	+19·2699	+19·3934	+19·5169	+19·6405	+19·7640
+410	+12·7100	+19·0650	+19·1921	+19·3192	+19·4463	+19·5734	+19·7005	+19·8276	+19·9547	+20·0818	+20·2089	+20·3360
+415	+13·0725	+19·6087	+19·7395	+19·8702	+20·0009	+20·1316	+20·2624	+20·3931	+20·5238	+20·6545	+20·7853	+20·9160
+420	+13·4400	+20·1600	+20·2944	+20·4288	+20·5632	+20·6976	+20·8320	+20·9664	+21·1008	+21·2352	+21·3696	+21·5040
+425	+13·8125	+20·7187	+20·8569	+20·9950	+21·1331	+21·2712	+21·4094	+21·5475	+21·6856	+21·8237	+21·9619	+22·1000
+430	+14·1900	+21·2850	+21·4269	+21·5688	+21·7107	+21·8526	+21·9945	+22·1364	+22·2783	+22·4202	+22·5621	+22·7040
+435	+14·5725	+21·8587	+22·0045	+22·1502	+22·2959	+22·4416	+22·5874	+22·7331	+22·8788	+23·0245	+23·1703	+23·3160
+440	+14·9600	+22·4400	+22·5896	+22·7392	+22·8888	+23·0384	+23·1880	+23·3376	+23·4872	+23·6368	+23·7864	+23·9360
+444·53	+15·3154	+22·9731	+23·1262	+23·2794	+23·4325	+23·5857	+23·7389	+23·8920	+24·0452	+24·1983	+24·3515	+24·5046
+445	+15·3525	+23·0287	+23·1823	+23·3358	+23·4893	+23·6428	+23·7964	+23·9499	+24·1034	+24·2569	+24·4105	+24·5640
+445·27	+15·3738	+23·0607	+23·2144	+23·3682	+23·5219	+23·6756	+23·8294	+23·9831	+24·1369	+24·2906	+24·4443	+24·5981
+450	+15·7500	+23·6250	+23·7825	+23·9400	+24·0975	+24·2550	+24·4125	+24·5700	+24·7275	+24·8850	+25·0425	+25·2000
+455	+16·1525	+24·2287	+24·3903	+24·5518	+24·7133	+24·8748	+25·0364	+25·1979	+25·3594	+25·5209	+25·6825	+25·8440
+460	+16·5600	+24·8400	+25·0056	+25·1712	+25·3368	+25·5024	+25·6680	+25·8336	+25·9992	+26·1648	+26·3304	+26·4960

APPENDIX TABLE II.—For calculating T when pt and δ are known, employing the formula $T = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}}$.

pt.	T. $\delta = 1{\cdot}54$.	$\delta = 1{\cdot}55$.	$\delta = 1{\cdot}56$.	$\delta = 1{\cdot}57$.	
−50	−48·879	−48·872	−48·865	−48·858	970
−49	−47·909	−47·902	−47·895	−47·888	
−48	−46·938	−46·931	−46·925	−46·918	1 97
−47	−45·967	−45·960	−45·954	−45·948	2 194
−46	−44·996	−44·989	−44·983	−44·977	3 291
−45	−44·024	−44·018	−44·012	−44·006	4 388
−44	−43·052	−43·046	−43·040	−43·034	5 485
−43	−42·079	−42·074	−42·068	−42·062	6 582
−42	−41·106	−41·101	−41·096	−41·090	7 679
−41	−40·133	−40·128	−40·123	−40·118	8 776
−40	−39·160	−39·155	−39·150	−39·145	9 873
−39	−38·187	−38·182	−38·177	−38·172	
−38	−37·214	−37·209	−37·204	−37·199	
−37	−36·240	−36·235	−36·230	−36·225	
−36	−35·266	−35·261	−35·256	−35·251	
−35	−34·291	−34·287	−34·282	−34·277	
−34	−33·316	−33·312	−33·308	−33·303	
−33	−32·341	−32·337	−32·333	−32·329	
−32	−31·366	−31·362	−31·358	−31·354	
−31	−30·390	−30·386	−30·382	−30·378	
−30	−29·414	−29·410	−29·406	−29·402	
−29	−28·438	−28·434	−28·430	−28·426	
−28	−27·461	−27·458	−27·454	−27·450	
−27	−26·484	−26·481	−26·478	−26·474	
−26	−25·507	−25·504	−25·501	−25·498	
−25	−24·530	−24·527	−24·524	−24·521	
−24	−23·552	−23·549	−23·546	−23·544	
−23	−22·574	−22·571	−22·568	−22·566	
−22	−21·596	−21·593	−21·590	−21·588	
−21	−20·617	−20·615	−20·612	−20·610	
−20	−19·638	−19·636	−19·634	−19·632	
−19	−18·659	−18·657	−18·655	−18·653	
−18	−17·680	−17·678	−17·676	−17·674	980
−17	−16·700	−16·698	−16·697	−16·695	
−16	−15·720	−15·718	−15·717	−15·716	1 98
−15	−14·740	−14·738	−14·737	−14·736	2 196
−14	−13·759	−13·758	−13·757	−13·756	3 294
−13	−12·778	−12·777	−12·776	−12·775	4 392
−12	−11·797	−11·796	−11·795	−11·794	5 490
−11	−10·816	−10·815	−10·814	−10·813	6 588
−10	−9·834	−9·833	−9·832	−9·831	7 686
−9	−8·852	−8·851	−8·850	−8·849	8 784
−8	−7·870	−7·869	−7·868	−7·867	9 882
−7	−6·887	−6·886	−6·886	−6·885	
−6	−5·904	−5·903	−5·903	−5·903	
−5	−4·921	−4·920	−4·920	−4·920	
−4	−3·937	−3·937	−3·937	−3·937	
−3	−2·953	−2·953	−2·953	−2·953	
−2	−1·969	−1·969	−1·969	−1·969	
−1	−0·985	−0·985	−0·985	−0·985	

APPENDIX TABLE II. (continued).—For calculating T when *pt* and δ are known, employing the formula $T = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}}$.

pt	T.			
	δ = 1·54.	δ = 1·55.	δ = 1·56.	δ = 1·57.
0	0·000	0·000	0·000	0·000
1	0·985	0·985	0·985	0·985
2	1·970	1·970	1·970	1·970
3	2·955	2·955	2·955	2·955
4	3·941	3·941	3·941	3·941
5	4·927	4·927	4·927	4·927
6	5·914	5·913	5·913	5·913
7	6·901	6·900	6·899	6·899
8	7·888	7·887	7·886	7·885
9	8·875	8·874	8·873	8·872
10	9·863	9·862	9·861	9·860
11	10·851	10·850	10·849	10·848
12	11·839	11·838	11·837	11·836
13	12·827	12·826	12·825	12·824
14	13·816	13·815	13·814	13·813
15	14·805	14·804	14·803	14·802
16	15·795	15·794	15·792	15·791
17	16·785	16·784	16·782	16·780
18	17·775	17·774	17·772	17·770
19	18·765	18·764	18·762	18·760
20	19·756	19·754	19·752	19·751
21	20·747	20·745	20·743	20·741
22	21·738	21·736	21·734	21·732
23	22·730	22·728	22·726	22·724
24	23·722	23·720	23·718	23·716
25	24·714	24·712	24·710	24·708
26	25·706	25·704	25·702	25·700
27	26·699	26·697	26·695	26·693
28	27·692	27·690	27·688	27·686
29	28·685	28·683	28·681	28·679
30	29·679	29·677	29·675	29·673
31	30·673	30·671	30·669	30·667
32	31·667	31·665	31·663	31·661
33	32·661	32·659	32·657	32·655
34	33·656	33·654	33·652	33·650
35	34·651	34·649	34·647	34·645
36	35·646	35·644	35·642	35·640
37	36·642	36·640	36·638	36·636
38	37·638	37·636	37·634	37·632
39	38·634	38·632	38·630	38·628
40	39·631	39·629	39·627	39·625
41	40·628	40·626	40·624	40·622
42	41·625	41·623	41·621	41·619
43	42·623	42·621	42·619	42·617
44	43·621	43·619	43·617	43·615
45	44·619	44·617	44·615	44·613
46	45·617	45·615	45·613	45·611
47	46·616	46·614	46·612	46·610
48	47·615	47·613	47·611	47·609
49	48·615	48·613	48·611	48·609

990	
1	99
2	198
3	297
4	396
5	495
6	594
7	693
8	792
9	891

1000	
1	100
2	200
3	300
4	400
5	500
6	600
7	700
8	800
9	900

Appendix Table II. (continued).—For calculating T when pt and δ are known, employing the formula $T = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}}$.

pt.	T.					
	$\delta = 1·54$.	$\delta = 1·55$.	$\delta = 1·56$.	$\delta = 1·57$.		
50	49·615	49·613	49·611	49·609	1000	
51	50·615	50·613	50·611	50·609		
52	51·615	51·613	51·611	51·609	1	100
53	52·616	52·614	52·612	52·610	2	200
54	53·617	53·615	53·613	53·611	3	300
55	54·619	54·617	54·615	54·613	4	400
56	55·621	55·619	55·617	55·615	5	500
57	56·623	56·621	56·619	56·617	6	600
58	57·625	57·623	57·621	57·619	7	700
59	58·627	58·625	58·623	58·621	8	800
60	59·630	59·627	59·625	59·623	9	900
61	60·633	60·630	60·628	60·626		
62	61·636	61·633	61·631	61·629		
63	62·639	62·637	62·635	62·633		
64	63·643	63·641	63·639	63·637		
65	64·648	64·646	64·644	64·642		
66	65·653	65·651	65·649	65·647		
67	66·658	66·656	66·654	66·652		
68	67·663	67·661	67·659	67·657		
69	68·669	68·667	68·665	68·663		
70	69·675	69·673	69·671	69·669		
71	70·681	70·679	70·677	70·675		
72	71·687	71·685	71·683	71·681		
73	72·694	72·692	72·690	72·688		
74	73·701	73·699	73·697	73·695		
75	74·709	74·707	74·705	74·703		
76	75·717	75·715	75·713	75·711		
77	76·725	76·723	76·721	76·719		
78	77·734	77·732	77·730	77·728		
79	78·743	78·741	78·739	78·737		
80	79·752	79·750	79·748	79·746		
81	80·761	80·759	80·757	80·755	1010	
82	81·771	81·769	81·767	81·765		
83	82·781	82·779	82·777	82·775	1	101
84	83·791	83·789	83·787	83·785	2	202
85	84·801	84·800	84·798	84·796	3	303
86	85·812	85·811	85·809	85·808	4	404
87	86·823	86·822	86·821	86·820	5	505
88	87·835	87·834	87·833	87·832	6	606
89	88·847	88·846	88·845	88·844	7	707
90	89·859	89·858	89·857	89·856	8	808
91	90·872	90·871	90·870	90·869	9	909
92	91·885	91·884	91·883	91·882		
93	92·898	92·897	92·896	92·895		
94	93·912	93·911	93·910	93·909		
95	94·926	94·925	94·924	94·924		
96	95·940	95·939	95·939	95·939		
97	96·954	96·954	96·954	96·954		
98	97·969	97·969	97·969	97·969		
99	98·984	98·984	98·984	98·984		

APPENDIX TABLE II. (continued).—For calculating T when pt and δ are known, employing the formula $T = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}}$.

pt.	T.					
	$\delta = 1{\cdot}54$	$\delta = 1{\cdot}55$.	$\delta = 1{\cdot}56$.	$\delta = 1{\cdot}57$.		
100	100·000	100·000	100·000	100·000		
101	101·016	101·016	101·016	101·016		
102	102·032	102·032	102·032	102·032		
103	103·048	103·049	103·049	103·049		
104	104·065	104·066	104·066	104·066		
105	105·082	105·083	105·083	105·084		
106	106·099	106·100	106·101	106·102		
107	107·117	107·118	107·119	107·120		
108	108·135	108·136	108·137	108·138		
109	109·154	109·155	109·156	109·157		
110	110·173	110·174	110·175	110·176		
111	111·192	111·193	111·194	111·195		
112	112·211	112·212	112·214	112·214	1020	
113	113·230	113·232	113·234	113·235		
114	114·250	114·252	114·254	114·256	1	102
115	115·271	115·273	115·275	115·277	2	204
116	116·292	116·294	116·296	116·298	3	306
117	117·313	117·315	117·317	117·320	4	408
118	118·334	118·336	118·339	118·342	5	510
119	119·355	119·358	119·361	119·364	6	612
120	120·377	120·380	120·383	120·386	7	714
121	121·400	121·403	121·406	121·409	8	816
122	122·423	122·426	122·429	122·432	9	918
123	123·446	123·449	123·452	123·455		
124	124·469	124·472	124·476	124·478		
125	125·492	125·496	125·500	125·502		
126	126·516	126·520	126·524	126·526		
127	127·540	127·544	127·548	127·551		
128	128·565	128·569	128·573	128·576		
129	129·590	129·594	129·598	129·602		
130	130·616	130·620	130·624	130·628		
131	131·642	131·646	131·650	131·654		
132	132·668	132·672	132·676	132·680		
133	133·694	133·698	133·703	133·707		
134	134·720	134·725	134·730	134·734		
135	135·747	135·752	135·757	135·762		
136	136·774	136·780	136·785	136·790		
137	137·802	137·808	137·813	137·818		
138	138·830	138·836	138·841	138·846		
139	139·858	139·864	139·870	139·875	1030	
140	140·887	140·893	140·899	140·904		
141	141·916	141·922	141·928	141·934	1	103
142	142·945	142·952	142·957	142·964	2	206
143	143·975	143·982	143·987	143·994	3	309
144	145·005	145·012	145·017	145·025	4	412
145	146·035	146·042	146·048	146·056	5	515
146	147·066	147·073	147·079	147·087	6	618
147	148·097	148·104	148·111	148·118	7	721
148	149·128	149·136	149·143	149·150	8	824
149	150·160	150·168	150·175	150·183	9	927

APPENDIX TABLE II. (continued).—For calculating T when pt and δ are known, employing the formula $T = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}}$.

pt.	T. $\delta = 1{\cdot}54$.	T. $\delta = 1{\cdot}55$.	T. $\delta = 1{\cdot}56$.	T. $\delta = 1{\cdot}57$.
150	151·192	151·200	151·208	151·216
151	152·224	152·232	152·241	152·249
152	153·257	153·265	153·274	153·282
153	154·290	154·299	154·308	154·316
154	155·324	155·333	155·342	155·350
155	156·358	156·367	156·376	156·384
156	157·392	157·401	157·410	157·418
157	158·426	158·435	158·444	158·453
158	159·460	159·470	159·479	159·488
159	160·495	160·505	160·515	160·524
160	161·531	161·541	161·551	161·560
161	162·567	162·577	162·587	162·597
162	163·603	163·613	163·623	163·634
163	164·639	164·650	164·660	164·672
164	165·676	165·687	165·697	165·710
165	166·713	166·724	166·735	166·748
166	167·750	167·762	167·773	167·786
167	168·788	168·800	168·811	168·824
168	169·826	169·838	169·850	169·862
169	170·864	170·877	170·889	170·901
170	171·903	171·916	171·929	171·940
171	172·942	172·956	172·969	172·980
172	173·982	173·996	174·009	174·021
173	175·022	175·036	175·049	175·063
174	176·062	176·076	176·090	176·105
175	177·102	177·117	177·131	177·147
176	178·143	178·158	178·173	178·189
177	179·185	179·200	179·215	179·231
178	180·227	180·242	180·257	180·273
179	181·269	181·284	181·300	181·316
180	182·311	182·327	182·343	182·359
181	183·353	183·370	183·386	183·403
182	184·396	184·413	184·430	184·447
183	185·440	185·457	185·474	185·491
184	186·484	186·501	186·518	186·536
185	187·528	187·545	187·563	187·581
186	188·572	188·590	188·608	188·626
187	189·617	189·635	189·653	189·671
188	190·662	190·680	190·698	190·716
189	191·707	191·726	191·744	191·763
190	192·753	192·772	192·790	192·810
191	193·799	193·818	193·837	193·857
192	194·845	194·865	194·884	194·904
193	195·892	195·912	195·932	195·952
194	196·939	196·960	196·980	197·000
195	197·987	198·008	198·028	198·049
196	199·035	199·056	199·077	199·098
197	200·084	200·105	200·126	200·147
198	201·133	201·154	201·175	201·196
199	202·182	202·203	202·225	202·246

1040	
1	104
2	208
3	312
4	416
5	520
6	624
7	728
8	832
9	936

APPENDIX TABLE II. (continued).—For calculating T when *pt* and δ are known, employing the formula $T = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}}$.

pt.	T.			
	δ = 1·54.	δ = 1·55.	δ = 1·56.	δ = 1·57.
200	203·231	203·253	203·275	203·296
201	204·281	204·303	204·325	204·347
202	205·331	205·353	205·376	205·399
203	206·381	206·404	206·427	206·451
204	207·431	207·455	207·479	207·503
205	208·482	208·507	208·531	208·555
206	209·534	209·559	209·583	209·607
207	210·586	210·611	210·635	210·659
208	211·638	211·663	211·688	211·712
209	212·690	212·716	212·742	212·766
210	213·743	213·769	213·796	213·820
211	214·796	214·823	214·850	214·875
212	215·850	215·877	215·904	215·930
213	216·904	216·931	216·958	216·985
214	217·958	217·986	218·013	218·041
215	219·013	219·041	219·069	219·097
216	220·068	220·097	220·125	220·153
217	221·124	221·153	221·182	221·210
218	222·180	222·209	222·239	222·267
219	223·236	223·266	223·296	223·325
220	224·293	224·323	224·353	224·383
221	225·350	225·380	225·410	225·441
222	226·407	226·438	226·468	226·499
223	227·464	227·496	227·527	227·557
224	228·522	228·554	228·586	228·616
225	229·581	229·613	229·645	229·676
226	230·640	230·672	230·704	230·736
227	231·699	231·731	231·764	231·797
228	232·758	232·791	232·824	232·858
229	233·817	233·851	233·885	233·919
230	234·877	234·912	234·946	234·980
231	235·938	235·973	236·007	236·041
232	236·999	237·035	237·069	237·103
233	238·061	238·097	238·131	238·165
234	239·123	239·159	239·194	239·228
235	240·185	240·221	240·257	240·291
236	241·248	241·284	241·320	241·355
237	242·311	242·347	242·384	242·420
238	243·374	243·411	243·448	243·485
239	244·437	244·475	244·512	244·550
240	245·501	245·539	245·577	245·615
241	246·565	246·604	246·642	246·680
242	247·630	247·669	247·707	247·746
243	248·695	248·734	248·773	248·812
244	249·760	249·800	249·839	249·879
245	250·826	250·866	250·906	250·946
246	251·892	251·933	251·973	252·014
247	252·958	253·000	253·041	253·082
248	254·025	254·067	254·109	254·150
249	255·092	255·135	255·177	255·219

1050	
1	105
2	210
3	315
4	420
5	525
6	630
7	735
8	840
9	945

1060	
1	106
2	212
3	318
4	424
5	530
6	636
7	742
8	848
9	954

Appendix Table II. (continued).—For calculating T when pt and δ are known, employing the formula $T = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}}$

pt.	T.				
	$\delta = 1·54$.	$\delta = 1·55$.	$\delta = 1·56$.	$\delta = 1·57$.	
250	256·160	256·203	256·245	256·288	
251	257·228	257·271	257·314	257·357	
252	258·296	258·340	258·383	258·427	
253	259·365	259·409	259·453	259·497	1070
254	260·434	260·479	260·523	260·568	
255	261·504	261·549	261·594	261·639	1 107
256	262·574	262·620	262·665	262·711	2 214
257	263·644	263·691	263·737	263·783	3 321
258	264·715	264·762	264·809	264·855	4 428
259	265·786	265·833	265·881	265·928	5 535
260	266·857	266·905	266·953	267·001	6 642
261	267·929	267·977	268·025	268·074	7 749
262	269·001	269·050	269·098	269·148	8 856
263	270·073	270·123	270·172	270·222	9 963
264	271·146	271·196	271·246	271·296	
265	272·219	272·270	272·320	272·370	
266	273·293	273·344	273·395	273·445	
267	274·367	274·419	274·470	274·521	
268	275·442	275·494	275·545	275·597	
269	276·517	276·569	276·621	276·673	
270	277·592	277·645	277·698	277·750	
271	278·667	278·721	278·775	278·827	
272	279·743	279·798	279·852	279·905	
273	280·820	280·875	280·930	280·984	
274	281·897	281·952	282·008	282·063	
275	282·974	283·029	283·086	283·142	
276	284·052	284·107	284·164	284·221	
277	285·130	285·186	285·243	285·300	
278	286·209	286·265	286·323	286·380	
279	287·288	287·344	287·403	287·460	1080
280	288·367	288·424	288·483	288·541	
281	289·446	289·504	289·563	289·622	1 108
282	290·525	290·584	290·644	290·704	2 216
283	291·605	291·665	291·726	291·786	3 324
284	292·685	292·746	292·808	292·868	4 432
285	293·766	293·827	293·890	293·951	5 540
286	294·847	294·909	294·972	295·034	6 648
287	295·929	295·992	296·055	296·117	7 756
288	297·012	297·075	297·139	297·201	8 864
289	298·095	298·158	298·223	298·286	9 972
290	299·178	299·241	299·307	299·371	
291	300·261	300·325	300·391	300·456	
292	301·344	301·409	301·476	301·541	
293	302·428	302·494	302·561	302·627	
294	303·512	303·579	303·646	303·713	
295	304·597	304·665	304·732	304·800	
296	305·682	305·751	305·819	305·887	
297	306·768	306·837	306·906	306·975	
298	307·854	307·924	307·993	308·063	
299	308·940	309·011	309·080	309·151	

Appendix Table II. (continued).—For calculating T when pt and δ are known, employing the formula $T = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}}$.

pt.	T.					
	$\delta = 1·54$.	$\delta = 1·55$.	$\delta = 1·56$.	$\delta = 1·57$.		
300	310·027	310·098	310·168	310·240		
301	311·115	311·186	311·257	311·329		
302	312·203	312·274	312·346	312·419		
303	313·290	313·363	313·435	313·509	1090	
304	314·379	314·452	314·525	314·599		
305	315·468	315·542	315·616	315·690	1	109
306	316·557	316·632	316·707	316·781	2	218
307	317·647	317·722	317·798	317·873	3	327
308	318·737	318·813	318·889	318·965	4	436
309	319·827	319·904	319·980	320·057	5	545
310	320·918	320·995	321·072	321·150	6	654
311	322·010	322·087	322·165	322·243	7	763
312	323·102	323·180	323·258	323·337	8	872
313	324·194	324·273	324·352	324·432	9	981
314	325·286	325·366	325·446	325·527		
315	326·379	326·459	326·540	326·622		
316	327·472	327·553	327·634	327·717		
317	328·565	328·647	328·729	328·812		
318	329·659	329·742	329·824	329·908		
319	330·754	330·837	330·920	331·004		
320	331·849	331·933	332·017	332·101		
321	332·944	333·029	333·114	333·199		
322	334·039	334·125	334·211	334·297		
323	335·135	335·222	335·308	335·395		
324	336·232	336·319	336·406	336·494		
325	337·329	337·417	337·505	337·593		
326	338·426	338·515	338·604	338·693		
327	339·524	339·613	339·703	339·793		
328	340·622	340·712	340·802	340·893		
329	341·720	341·811	341·902	341·993	1100	
330	342·819	342·911	343·003	343·094		
331	343·918	344·011	344·104	344·196	1	110
332	345·018	345·112	345·205	345·298	2	220
333	346·119	346·213	346·307	346·401	3	330
334	347·220	347·314	347·409	347·504	4	440
335	348·321	348·416	348·511	348·607	5	550
336	349·422	349·518	349·614	349·710	6	660
337	350·523	350·620	350·717	350·814	7	770
338	351·625	351·723	351·820	351·918	8	880
339	352·728	352·826	352·924	353·022	9	990
340	353·831	353·930	354·029	354·127		
341	354·934	355·034	355·134	355·233		
342	356·038	356·139	356·239	356·340		
343	357·143	357·244	357·345	357·447		
344	358·248	358·350	358·452	358·554		
345	359·353	359·456	359·559	359·662		
346	360·458	360·562	360·666	360·770		
347	361·564	361·668	361·773	361·878		
348	362·670	362·775	362·880	362·986		
349	363·777	363·883	363·988	364·095		

APPENDIX TABLE II. (continued).—For calculating T when *pt* and δ are known, employing the formula $T = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}}$.

pt.	T. δ = 1·54.	δ = 1·55.	δ = 1·56.	δ = 1·57.
350	364·884	364·991	365·097	365·205
351	365·992	366·100	366·207	366·316
352	367·100	367·209	367·317	367·427
353	368·208	368·318	368·427	368·538
354	369·317	369·428	369·538	369·649
355	370·426	370·538	370·649	370·760
356	371·536	371·648	371·760	371·871
357	372·647	372·759	372·871	372·983
358	373·758	373·871	373·984	374·097
359	374·869	374·983	375·097	375·211
360	375·980	376·095	376·210	376·325
361	377·092	377·208	377·324	377·440
362	378·204	378·321	378·438	378·555
363	379·316	379·434	379·552	379·670
364	380·429	380·548	380·666	380·785
365	381·543	381·662	381·781	381·900
366	382·657	382·777	382·897	383·017
367	383·772	383·892	384·013	384·134
368	384·887	385·008	385·129	385·251
369	386·002	386·124	386·246	386·369
370	387·118	387·241	387·364	387·488
371	388·234	388·358	388·482	388·607
372	389·350	389·475	389·600	389·726
373	390·467	390·593	390·719	390·845
374	391·584	391·711	391·838	391·965
375	392·702	392·830	392·958	393·086
376	393·820	393·949	394·078	394·207
377	394·939	395·069	395·199	395·329
378	396·058	396·189	396·320	396·451
379	397·177	397·309	397·441	397·573
380	398·297	398·430	398·563	398·696
381	399·417	399·551	399·685	399·819
382	400·538	400·673	400·808	400·943
383	401·659	401·795	401·931	402·067
384	402·781	402·918	403·055	403·192
385	403·903	404·041	404·179	404·317
386	405·026	405·165	405·304	405·443
387	406·149	406·289	406·429	406·569
388	407·272	407·413	407·554	407·695
389	408·396	408·538	408·679	408·822
390	409·521	409·663	409·805	409·949
391	410·646	410·789	410·932	411·076
392	411·771	411·915	412·060	412·204
393	412·896	413·041	413·187	413·333
394	414·021	414·168	414·315	414·462
395	415·148	415·296	415·444	415·592
396	416·275	416·424	416·573	416·722
397	417·402	417·552	417·702	417·852
398	418·530	418·681	418·832	418·983
399	419·658	419·810	419·962	420·114

1110	
1	111
2	222
3	333
4	444
5	555
6	666
7	777
8	888
9	999

1120	
1	112
2	224
3	336
4	448
5	560
6	672
7	784
8	896
9	1008

Appendix Table II. (continued).—For calculating T when pt and δ are known, employing the formula $T = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}}$.

pt.	T. $\delta = 1{\cdot}54$.	T. $\delta = 1{\cdot}55$.	T. $\delta = 1{\cdot}56$.	T. $\delta = 1{\cdot}57$.	
400	420·787	420·940	421·093	421·246	1130
401	421·916	422·070	422·224	422·378	
402	423·046	423·201	423·356	423·511	1 113
403	424·176	424·332	424·488	424·644	2 226
404	425·306	425·463	425·620	425·777	3 339
405	426·437	426·595	426·753	426·911	4 452
406	427·569	427·728	427·887	428·046	5 565
407	428·701	428·861	429·021	429·181	6 678
408	429·833	429·994	430·055	430·216	7 791
409	430·966	431·128	431·290	431·452	8 904
410	432·099	432·262	432·425	432·588	9 1017
411	433·233	433·397	433·561	433·725	
412	434·367	434·532	434·697	434·862	
413	435·501	435·667	435·833	435·999	
414	436·636	436·803	436·970	437·137	
415	437·772	437·940	438·108	438·276	
416	438·908	439·077	439·246	439·416	
417	440·044	440·214	440·385	440·556	
418	441·181	441·352	441·524	441·696	
419	442·318	442·490	442·663	442·836	
420	443·456	443·629	443·803	443·977	
421	444·594	444·768	444·943	445·118	
422	445·733	445·908	446·084	446·260	1140
423	446·872	447·048	447·225	447·402	
424	448·011	448·188	448·366	448·544	1 114
425	449·151	449·329	449·508	449·687	2 228
426	450·291	450·471	450·650	450·831	3 342
427	451·432	451·613	451·794	451·976	4 456
428	452·573	452·755	452·938	453·121	5 570
429	453·715	453·898	454·082	454·266	6 684
430	454·858	455·042	455·227	455·412	7 798
431	456·001	456·186	456·372	456·558	8 912
432	457·144	457·330	457·517	457·704	9 1026
433	458·287	458·475	458·663	458·851	
434	459·431	459·620	459·810	459·999	
435	460·576	460·766	460·957	461·147	
436	461·721	461·912	462·104	462·296	
437	462·866	463·058	463·252	463·445	
438	464·011	464·205	464·400	464·595	
439	465·158	465·353	465·549	465·745	1150
440	466·305	466·501	466·698	466·895	
441	467·452	467·649	467·847	468·045	1 115
442	468·600	468·798	468·997	469·196	2 230
443	469·748	469·948	470·148	470·348	3 345
444	470·897	471·098	471·299	471·500	4 460
445	472·046	472·248	472·451	472·653	5 575
446	473·196	473·399	473·603	473·807	6 690
447	474·346	474·550	474·756	474·961	7 805
448	475·496	475·702	475·909	476·115	8 920
449	476·647	476·854	477·062	477·270	9 1035

APPENDIX TABLE II. (continued).—For calculating T when pt and δ are known, employing the formula $T = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10000pt}{\delta}}$.

pt.	T.			
	$\delta = 1{\cdot}54$.	$\delta = 1{\cdot}55$.	$\delta = 1{\cdot}56$.	$\delta = 1{\cdot}57$.
450	477·799	478·007	478·216	478·425
451	478·951	479·160	479·370	479·580
452	480·103	480·314	480·525	480·736
453	481·256	481·468	481·680	481·893
454	482·410	482·623	482·836	483·050
455	483·564	483·778	483·992	484·207
456	484·718	484·933	485·149	485·365
457	485·873	486·089	486·306	486·524
458	487·028	487·246	487·464	487·683
459	488·184	488·403	488·622	488·842
460	489·341	489·561	489·781	490·002
461	490·498	490·719	490·940	491·162
462	491·655	491·877	492·100	492·323
463	492·812	493·036	493·260	493·485
464	493·969	494·195	494·421	494·647
465	495·127	495·355	495·582	495·810
466	496·286	496·516	496·744	496·973
467	497·446	497·677	497·907	498·137
468	498·606	498·838	499·070	499·301
469	499·766	500·000	500·233	500·465
470	500·927	501·162	501·396	501·630
471	502·089	502·325	502·560	502·796
472	503·251	503·488	503·725	503·962
473	504·414	504·652	504·890	505·128
474	505·577	505·816	506·055	506·295
475	506·741	506·981	507·221	507·462
476	507·905	508·147	508·388	508·630
477	509·070	509·313	509·555	509·799
478	510·235	510·479	510·723	510·968
479	511·400	511·646	511·891	512·138
480	512·565	512·812	513·060	513·308
481	513·731	513·981	514·229	514·478
482	514·898	515·149	515·399	515·649
483	516·065	516·318	516·569	516·821
484	517·233	517·487	517·740	517·993
485	518·402	518·657	518·911	519·165
486	519·571	519·827	520·082	520·338
487	520·741	520·998	521·254	521·512
488	521·911	522·169	522·427	522·686
489	523·081	523·340	523·600	523·861
490	524·251	524·512	524·773	525·036
491	525·422	525·685	525·947	526·211
492	526·594	526·858	527·122	527·387
493	527·766	528·032	528·297	528·563
494	528·938	529·206	529·473	529·740
495	530·111	530·381	530·649	530·917
496	531·285	531·556	531·825	532·095
497	532·460	532·732	533·002	533·274
498	533·635	533·908	534·179	534·453
499	534·811	535·085	535·358	535·633
500	535·987	536·262	536·537	536·813

1150	
1	115
2	230
3	345
4	460
5	575
6	690
7	805
8	920
9	1035

1160	
1	116
2	232
3	348
4	464
5	580
6	696
7	812
8	928
9	1044

1170	
1	117
2	234
3	351
4	468
5	585
6	702
7	819
8	936
9	1053

APPENDIX TABLE III.—Temperatures of Ebullition of Water under varying Pressures. Calculated by Dr. BROCH from the observations of REGNAULT.

Millims.	Tenths of a millimetre. 0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Pro. Pts.
730	98°·8802	·8840	·8878	·8916	·8954	·8992	·9030	·9068	·9106	·9144	38
731	·9182	·9220	·9258	·9295	·9333	·9371	·9409	·9447	·9485	·9523	1 3·8
732	·9561	·9599	·9637	·9674	·9712	·9750	·9788	·9826	·9864	·9902	2 7·6
733	·9939	·9977	·0015	·0053	·0091	·0128	·0166	·0204	·0242	·0280	3 11·4
734	99°·0318	·0355	·0393	·0431	·0469	·0506	·0544	·0582	·0620	·0658	4 15·2
735	99°·0695	·0733	·0771	·0808	·0846	·0884	·0922	·0959	·0997	·1035	5 19·0
736	·1073	·1110	·1148	·1186	·1223	·1261	·1299	·1336	·1374	·1412	6 22·8
737	·1449	·1487	·1525	·1562	·1600	·1638	·1675	·1713	·1751	·1788	7 26·6
738	·1826	·1863	·1901	·1939	·1976	·2014	·2051	·2089	·2127	·2164	8 30·4
739	·2202	·2239	·2277	·2315	·2352	·2390	·2427	·2465	·2502	·2540	9 34·2
740	99°·2577	·2615	·2652	·2690	·2728	·2765	·2803	·2840	·2878	·2915	37
741	·2953	·2990	·3028	·3065	·3102	·3140	·3177	·3215	·3252	·3290	1 3·7
742	·3327	·3365	·3402	·3440	·3477	·3514	·3552	·3589	·3627	·3664	2 7·4
743	·3702	·3739	·3776	·3814	·3851	·3889	·3926	·3963	·4001	·4038	3 11·1
744	·4075	·4113	·4150	·4187	·4225	·4262	·4299	·4337	·4374	·4412	4 14·8
745	99°·4449	·4486	·4523	·4561	·4598	·4635	·4673	·4710	·4747	·4785	5 18·5
746	·4822	·4859	·4896	·4934	·4971	·5008	·5045	·5083	·5120	·5157	6 22·2
747	·5194	·5232	·5269	·5306	·5343	·5381	·5418	·5455	·5492	·5529	7 25·9
748	·5567	·5604	·5641	·5678	·5715	·5752	·5790	·5827	·5864	·5901	8 29·6
749	·5938	·5975	·6013	·6050	·6087	·6124	·6161	·6198	·6235	·6273	9 33·3
750	99°·6310	·6347	·6384	·6421	·6458	·6495	·6532	·6569	·6606	·6643	36
751	·6681	·6718	·6755	·6792	·6829	·6866	·6903	·6940	·6977	·7014	1 3·6
752	·7051	·7088	·7126	·7162	·7199	·7236	·7273	·7310	·7347	·7384	2 7·2
753	·7421	·7458	·7495	·7532	·7569	·7606	·7643	·7680	·7717	·7754	3 10·8
754	·7791	·7828	·7865	·7902	·7938	·7975	·8012	·8049	·8086	·8123	4 14·4
755	99°·8160	·8197	·8234	·8271	·8308	·8344	·8381	·8418	·8455	·8492	5 18·0
756	·8529	·8566	·8603	·8639	·8676	·8713	·8750	·8787	·8824	·8860	6 21·6
757	·8897	·8934	·8971	·9008	·9044	·9081	·9118	·9155	·9192	·9228	7 25·2
758	·9265	·9302	·9339	·9376	·9412	·9449	·9486	·9523	·9559	·9596	8 28·8
759	·9633	·9670	·9706	·9743	·9780	·9816	·9853	·9890	·9927	·9964	9 32·4
760	100°·0000	·0037	·0073	·0110	·0147	·0183	·0220	·0257	·0293	·0330	36
761	·0367	·0403	·0440	·0477	·0513	·0550	·0587	·0623	·0660	·0696	1 3·6
762	·0733	·0770	·0806	·0843	·0880	·0916	·0953	·0989	·1026	·1062	2 7·2
763	·1099	·1136	·1172	·1209	·1245	·1282	·1318	·1355	·1392	·1428	3 10·8
764	·1465	·1501	·1538	·1574	·1611	·1647	·1684	·1720	·1757	·1793	4 14·4
765	100°·1830	·1866	·1903	·1939	·1976	·2012	·2049	·2085	·2122	·2158	5 18·0
766	·2194	·2231	·2267	·2304	·2340	·2377	·2413	·2450	·2486	·2522	6 21·6
767	·2559	·2595	·2632	·2668	·2704	·2741	·2777	·2814	·2850	·2886	7 25·2
768	·2923	·2959	·2995	·3032	·3068	·3105	·3141	·3177	·3214	·3250	8 28·8
769	·3286	·3323	·3359	·3395	·3432	·3468	·3504	·3540	·3577	·3613	9 32·4
770	100°·3649	·3686	·3722	·3758	·3794	·3831	·3867	·3903	·3940	·3976	

Appendix Table III. (continued).

Millims.	Tenths of a millimetre.										Pro. Pts.	
	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.		
770	100°·3649	·3686	·3722	·3758	·3794	·3831	·3867	·3903	·3940	·3976	36	
771	·4012	·4048	·4085	·4121	·4157	·4193	·4230	·4266	·4302	·4338		
772	·4374	·4411	·4447	·4483	·4519	·4555	·4592	·4628	·4664	·4700	1	3·6
773	·4736	·4773	·4809	·4845	·4881	·4917	·4953	·4989	·5026	·5062	2	7·2
774	·5098	·5134	·5170	·5206	·5242	·5278	·5315	·5351	·5387	·5423	3	10·8
											4	14·4
775	·5459	·5495	·5531	·5567	·5603	·5639	·5675	·5712	·5748	·5784	5	18·0
776	·5820	·5856	·5892	·5928	·5964	·6000	·6036	·6072	·6108	·6144	6	21·6
777	·6180	·6216	·6252	·6288	·6324	·6360	·6396	·6432	·6468	·6504	7	25·2
778	·6540	·6576	·6612	·6648	·6684	·6720	·6756	·6792	·6828	·6864	8	28·8
779	·6900	·6936	·6971	·7007	·7043	·7079	·7115	·7151	·7187	·7223	9	32·4

Appendix Table IV.—For Conversion of Water Pressure into its Equivalent in Mercury. 1 millim. Water = 0·0736 millim. Mercury.

Interpolated from table given in Landolt and Börnstein's 'Physikalisch-chemische Tabellen.'

Note.—Large temperature differences might influence the last figure of this table.

Water.	Mercury.	Water.	Mercury.
millims.	millims.	millims.	millims.
0·05	0·004	1·55	0·114
0·10	0·007	1·60	0·118
0·15	0·011	1·65	0·121
0·20	0·015	1·70	0·125
0·25	0·018	1·75	0·129
0·30	0·022	1·80	0·132
0·35	0·026	1·85	0·136
0·40	0·029	1·90	0·140
0·45	0·033	1·95	0·144
0·50	0·037	2·00	0·147
0·55	0·040	2·05	0·151
0·60	0·044	2·10	0·155
0·65	0·048	2·15	0·158
0·70	0·052	2·20	0·162
0·75	0·055	2·25	0·166
0·80	0·059	2·30	0·169
0·85	0·063	2·35	0·173
0·90	0·066	2·40	0·177
0·95	0·070	2·45	0·180
1·00	0·074	2·50	0·184
1·05	0·077	2·55	0·188
1·10	0·081	2·60	0·191
1·15	0·085	2·65	0·195
1·20	0·088	2·70	0·199
1·25	0·092	2·75	0·202
1·30	0·096	2·80	0·206
1·35	0·099	2·85	0·210
1·40	0·103	2·90	0·213
1·45	0·107	2·95	0·217
1·50	0·110	3·00	0·221

Appendix Table V.—Showing the value of $d \equiv T - pt$ corresponding to different values of T near 100°, deduced from the formula $d = \delta\left[\left(\frac{T}{100}\right)^2 - \frac{T}{100}\right]$.

Note.—Between 0° and 100° pt is greater than T, and d is negative. Above 100° pt is less than T, and d is positive.

T.	$\left(\frac{T}{100}\right)^2 - \frac{T}{100}$.	$\delta = 1·500$.	$\delta = 1·510$.	$\delta = 1·520$.	$\delta = 1·530$.	$\delta = 1·540$.	$\delta = 1·550$.	$\delta = 1·560$.	$\delta = 1·570$.	$\delta = 1·580$.	$\delta = 1·590$.	$\delta = 1·600$.
101·0	+0·0101	+0·0151	+0·0153	+0·0154	+0·0155	+0·0156	+0·0157	+0·0158	+0·0159	+0·0160	+0·0161	+0·0162
100·9	+0·0091	+0·0136	+0·0137	+0·0138	+0·0139	+0·0140	+0·0141	+0·0142	+0·0143	+0·0144	+0·0145	+0·0146
100·8	+0·0081	+0·0121	+0·0122	+0·0123	+0·0124	+0·0125	+0·0126	+0·0126	+0·0127	+0·0128	+0·0129	+0·0130
100·7	+0·0071	+0·0106	+0·0107	+0·0108	+0·0109	+0·0109	+0·0110	+0·0111	+0·0111	+0·0112	+0·0113	+0·0114
100·6	+0·0061	+0·0091	+0·0092	+0·0093	+0·0093	+0·0094	+0·0095	+0·0095	+0·0096	+0·0096	+0·0097	+0·0098
100·5	+0·0050	+0·0075	+0·0075	+0·0076	+0·0076	+0·0077	+0·0077	+0·0078	+0·0078	+0·0079	+0·0079	+0·0080
100·4	+0·0040	+0·0060	+0·0060	+0·0061	+0·0061	+0·0062	+0·0062	+0·0062	+0·0063	+0·0063	+0·0064	+0·0064
100·3	+0·0030	+0·0045	+0·0045	+0·0046	+0·0046	+0·0046	+0·0046	+0·0047	+0·0047	+0·0047	+0·0048	+0·0048
100·2	+0·0020	+0·0030	+0·0030	+0·0030	+0·0031	+0·0031	+0·0031	+0·0031	+0·0031	+0·0032	+0·0032	+0·0032
100·1	+0·0010	+0·0015	+0·0015	+0·0015	+0·0015	+0·0015	+0·0015	+0·0016	+0·0016	+0·0016	+0·0016	+0·0016
100·0	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000
99·9	−0·0010	−0·0015	−0·0015	−0·0015	−0·0015	−0·0015	−0·0015	−0·0016	−0·0016	−0·0016	−0·0016	−0·0016
99·8	−0·0020	−0·0030	−0·0030	−0·0030	−0·0031	−0·0031	−0·0031	−0·0031	−0·0031	−0·0032	−0·0032	−0·0032
99·7	−0·0030	−0·0045	−0·0045	−0·0046	−0·0046	−0·0046	−0·0046	−0·0047	−0·0047	−0·0047	−0·0048	−0·0048
99·6	−0·0040	−0·0060	−0·0060	−0·0061	−0·0061	−0·0062	−0·0062	−0·0062	−0·0063	−0·0063	−0·0064	−0·0064
99·5	−0·0049	−0·0073	−0·0074	−0·0074	−0·0075	−0·0075	−0·0076	−0·0076	−0·0077	−0·0077	−0·0078	−0·0078
99·4	−0·0059	−0·0088	−0·0089	−0·0090	−0·0090	−0·0091	−0·0091	−0·0092	−0·0093	−0·0093	−0·0094	−0·0094
99·3	−0·0069	−0·0103	−0·0104	−0·0105	−0·0106	−0·0106	−0·0107	−0·0108	−0·0108	−0·0109	−0·0110	−0·0110
99·2	−0·0079	−0·0118	−0·0119	−0·0120	−0·0121	−0·0122	−0·0122	−0·0123	−0·0124	−0·0125	−0·0126	−0·0126
99·1	−0·0089	−0·0133	−0·0134	−0·0135	−0·0136	−0·0137	−0·0138	−0·0139	−0·0140	−0·0141	−0·0142	−0·0142
99·0	−0·0099	−0·0148	−0·0149	−0·0150	−0·0151	−0·0152	−0·0153	−0·0154	−0·0155	−0·0156	−0·0157	−0·0158
98·9	−0·0109	−0·0163	−0·0164	−0·0166	−0·0167	−0·0168	−0·0169	−0·0170	−0·0171	−0·0172	−0·0173	−0·0174
98·8	−0·0118	−0·0177	−0·0178	−0·0179	−0·0181	−0·0182	−0·0183	−0·0184	−0·0185	−0·0186	−0·0188	−0·0189
98·7	−0·0128	−0·0192	−0·0193	−0·0195	−0·0196	−0·0197	−0·0198	−0·0200	−0·0201	−0·0202	−0·0204	−0·0205
98·6	−0·0138	−0·0207	−0·0208	−0·0210	−0·0211	−0·0213	−0·0214	−0·0215	−0·0217	−0·0218	−0·0219	−0·0221
98·5	−0·0147	−0·0220	−0·0222	−0·0223	−0·0225	−0·0226	−0·0228	−0·0229	−0·0231	−0·0232	−0·0234	−0·0235
98·4	−0·0157	−0·0235	−0·0237	−0·0239	−0·0240	−0·0242	−0·0243	−0·0245	−0·0246	−0·0248	−0·0250	−0·0251
98·3	−0·0167	−0·0250	−0·0252	−0·0254	−0·0256	−0·0257	−0·0259	−0·0261	−0·0262	−0·0264	−0·0266	−0·0267
98·2	−0·0177	−0·0265	−0·0267	−0·0269	−0·0271	−0·0273	−0·0274	−0·0276	−0·0278	−0·0280	−0·0281	−0·0283
98·1	−0·0186	−0·0279	−0·0281	−0·0283	−0·0285	−0·0286	−0·0288	−0·0290	−0·0292	−0·0294	−0·0296	−0·0298
98·0	−0·0196	−0·0294	−0·0296	−0·0298	−0·0300	−0·0302	−0·0304	−0·0306	−0·0308	−0·0310	−0·0312	−0·0314

APPENDIX II. [Added December 1, 1899.]

We think we are justified in adding, in the form of an appendix, some further considerations on the question of the sulphur boiling-point, the results of which we obtained since the date of handing in the paper. On p. 99 of the text are given the observations of REGNAULT on the variation of the boiling-point of sulphur with pressure near 760 millims. The formula used by REGNAULT himself to express the results of his observations over the whole range was of a logarithmic kind, and gave for the pressure 760 millims. the value 448°·38. If, however, we disregard the extreme portions of the range and find a formula to represent only those observations near the normal pressure, we find for this point a value nearly a degree lower.

Taking the four observations quoted in the text, and representing them by a formula

$$T = a + bp + cp^2,$$

of which the constants a, b, and c are determined by least squares, we find for $p = 760$ the value 447°·51, with residuals very much smaller than those given by the logarithmic formula. We are aware that to represent four observations by a formula with three constants is not giving very much latitude for probable errors, but we think, nevertheless, that 447°·5 gives much more nearly the true result to be deduced from REGNAULT'S experiments than his own much higher figure. Accepting this method of treating his observations, we further find that instead of the value for dt/dp 0°·082 per millim. as given by the logarithmic formula, we get 0°·088, a value very appreciably higher.

If our determinations of pt_s had all been made at 760 millims. pressure, or if this had been the mean pressure of each different series, the value to be taken for dt/dp would have been of no great consequence, but as in each case the mean pressure fell appreciably below this, we thought it desirable to see how much the assumption of the higher value might influence the results of our experiments.

We gave in the text the results of some calculations on the series of sulphur points taken with the Kew platinum thermometers K.1 and K.3, made with the object of arriving at an independent value for dt/dp. Dr. CHREE has recently completed for publication an investigation into the behaviour of the Kew platinum thermometers, and their permanence over a considerable period, and finds that, when one or two sources of uncertainty are eliminated, the values we gave for dt/dp for K.1 and K.3 are both somewhat too small. He has courteously permitted us to state that the most probable value for this number deducible from the different series of determinations of the sulphur point, which he has worked up, is much more nearly

0°·090 than 0°·082, agreeing in a remarkable manner with the result we have just deduced from REGNAULT'S experiments.

As the mean pressure of our sulphur point determinations was below 760 millims. in all the series, we thought it of interest to recalculate the results of each set, applying the value 0·088 for dt/dp. Combining this with the known value of $d\,.\,pt/dt$, we have for $d\,.\,pt/dp$ the value 0·0773 at 445°.

The values of pt_s from the separate experiments with K.8 and K.9, are given in the following table :—

K.8.	K.9.
421·58	421·46
·56	·42
·52	·49
·53	·49
·56	·44
·57	
·59	
421·559	421·460

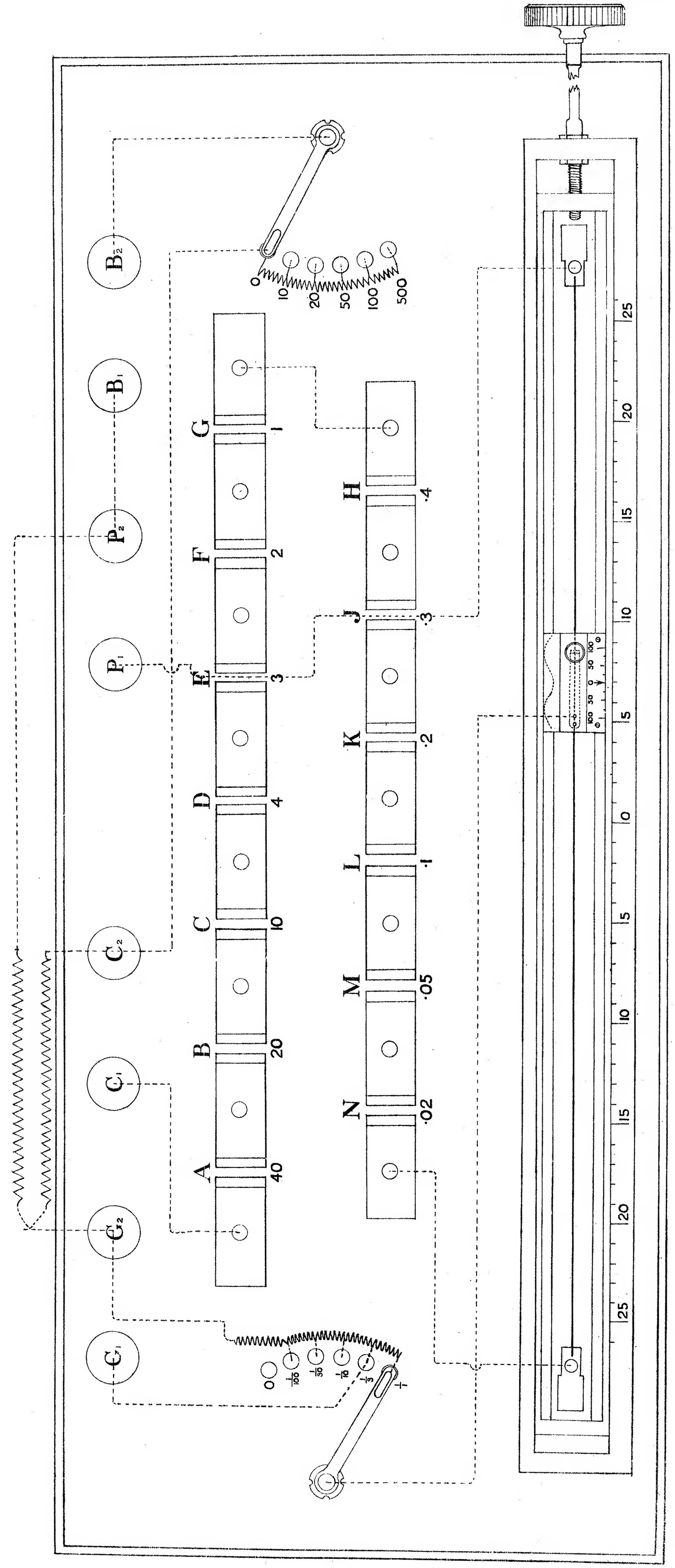
These values of pt_s only differ very slightly from those previously found.

We next proceed to find for each thermometer from the equivalent values of T and pt given by the comparisons near the sulphur point the T_s corresponding to the value of pt_s deduced above. We formerly used for this purpose a formula containing the term $(pt - pt_s)$ to the first and second powers, but as there appeared some doubt as to how the result might be affected by stopping short at the second term, in the new calculation we tried several formulæ of different types, and included varying numbers of experiments in the neighbourhood of the sulphur point.

We had already satisfied ourselves that CALLENDAR'S formula closely represents the divergence between the platinum and gas scales over the range covered by our experiments. Utilising this formula and including for K.8 all the experiments between T = 412°·65 and T = 455°·54, nine in all, we obtain for the T_s corresponding to the pt_s above given the value 445°·27, which is sensibly identical with that previously found. For the two series with K.9, however, we find that while the first series of observations gives a result for T_s 445°·27, the second series, including the comparisons between T = 405°·93 and T = 450°·58, gives 445°·05, which is appreciably lower than the result given in the text. The discrepancy between the two values furnished by the thermometer K.9 is lessened by excluding some of the comparisons which are at some distance from the sulphur point, but the mean result is hardly sensibly affected.

We have also made the same kind of calculation of a value for T_s from the comparisons with thermometer K.2, though, in this case, none of the comparisons were made at temperatures very near the sulphur point. We find, employing the same formula to obtain the T_s, the value 445°·1, which is only 0°·1 lower than the mean previously found from the K.8 and K.9 experiments. During this series of comparisons the pressure of the nitrogen in the gas thermometer was sensibly lower than in any of the preceding ones, being only 392 millims. instead of 529 millims.

In view of the uncertainties in the value of dt/dp and those arising from imperfect data as to the expansion of the porcelain at high temperatures, we prefer to suppress the hundredths of a degree from our mean result for the temperature of the boiling-point of sulphur, and to give for this point the value $T_s =$ **445°·2** *on the scale of the constant volume nitrogen thermometer.*



West, Newman photo-lith.